13: Discrete Probability Distributions

Key Terms

- **Bernoulli trial:** is an experiment whose outcome is random and can be either of two possible outcomes, "success" and "failure."
- **Independent trials**: the outcome from one trial has no effect on the outcome to be obtained from any other trial.
- **Binomial Experiment**: 1) predetermined n number of independent Bernoulli trials. 2) each trial result in only two outcomes: success and failure. 3) P(success)= p, is constant.
- **Geometric distribution:** probability of the number of times needed to do something until the first successful outcome. The number of Bernoulli trials which must be conducted before a trial results in a success.
- **Geometric sequence:** a sequence of numbers in which the $(n+1)^{th}$ number is a multiple of the n^{th} number. $P(X = n+1)$ is a multiple of $P(X = n)$.

Symbols

- X = Binomial or Geometric random variable
- x = number of successful trials
- p = probability of a success
- 1-p = probability of failure for a single trial
- μ = mean
- σ = standard deviation
- σ^2 = variance
- C = number of possible ways to have X = x
- k = number of trials needed for first success

Distribution Properties

- **Bernoulli distribution**:
  1. $\mu=px(1)+(1-p)x(0)=p$
  2. $\sigma^2=pq$
- **Binomial distribution:**
  1. $$P(X=x)=C_x^n p^x(1-p)^{n-x} = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x},$$ $$\text{where } x=0,1,2,...n,\ 0<p<1$$
  2. $\mu=np$
  3. $\sigma^2=npq=np(1-p)$
- **Geometric distribution:**
- Probability:

If the probability of success on each trial is *p*, then the probability that *k* trials are needed to get one success is either

$P(X=k)=(1-p)^{k-1}p$

for *k* = 1, 2, 3, ...,

- Mean: $E(X)= 1/p$
- Variance : $Var(X)=(1-p)/p^2$

Geometric Distribution Example

An experiment consists of rolling a single die. The event of interest is rolling a 2; this event is called a success. Is this a geometric experiment?

- Rolling a 2 will represent a success, and rolling any other number will represent a failure.
- The probability of rolling a 2 on each roll is the same, p = 1/6.
- The observations are independent.
- A trial consists of rolling the die once. We roll the die until the first 2 appears.

Since all of the requirements are satisfied, **this experiment describes a geometric setting**

Binomial Distribution Example

A random sample of 15 men is selected and the number who voted for Bush is recorded. Is this an example of a binomial experiment?

- n independent and identical trials:
  - Trials: all men
  - n=15
- Two outcomes, Success and Failure:
  - Success=voted for Bush
  - Failure=did not vote for Bush
  - Probability of success and failure:
    - P(S)=0.4
    - P(F)=0.6
  - x is the number of successes:
    - x= number of men who voted for Bush

Yes, this is a binomial experiment

Comparison of Geometric and Binomial

- The geometric distribution is the only discrete memoryless random distribution.
- It is a discrete analog of the exponential distribution.
- This means that the chance of getting a heads up on the 7^{th} trial after failing the first 6 times is the same probability as getting a heads on any of the first 6 trials.
- The random process does not "remember" the number of failures.
- In the *binomial distribution* we have fixed number of trials and a variable number of successes
- In the *geometric distribution* we wait for a single success, but the number of trials is variable.
- *Negative Binomial distribution* is the sum of Geometric distribution. How many trials will be needed to have the first "x" number of successes.
  - If *Y1,...,Yr* are independent geometrically distributed variables with parameter *p*, then
    $$Z=\sum_{m=1}^{r} Y_m$$
  - Follows a negative binomial distribution with parameters *r* and *p*.

Calculating Binomial Probability

A student claims that he gets grades better or equal to A, 40% of the time. This quarter, he gets only one A out of 4 courses. How likely is it that he got one A, or worse, out of four courses given his claim?

$$P(x=0)=\frac{4!}{0!(4!)}(0.4)^0(0.6)^4=1(1)(0.006)=0.1296$$

$$P(x=1)=\frac{4!}{1!(3!)}(0.4)^1(0.6)^3=4(0.4)(0.216)=0.3456$$

$$P(x=0)+P(x=1)=0.5616$$

Calculating Geometric Probability

A child is trying to pick a yellow marble from a jar of 10 marbles with replacement and only 3 red marble in the jar. What is the probability of the girl succeeding in the 6th trial?

P=0.3

$P(X=6)=0.3(1-0.3)^{6-1}$ = **0.050**

How to Use This Cheat Sheet: These are the keys related this topic. Try to read through it carefully twice then write it out on a blank sheet of paper. Review it again before the exams.