09: Factoring Polynomials

Key Terms

- **Binomial:** a polynomial with exactly two terms.
- **Factorable polynomial:** a polynomial that can be converted to the product of some other polynomials.
- **Factoring:** the process of finding polynomials whose product is equivalent to a given polynomial.
- **Monomial:** a polynomial with exactly one term.
- **Polynomial:** the sum of terms with real coefficients and variable factors with whole number exponents; the sum of monomials.
- **Prime polynomial:** a polynomial that cannot be converted to the product of at least two factors other than 1.
- **Trinomial:** a polynomial with exactly three terms.

Special Factoring Identities

- **Square of a Binomial – Addition**

$$a^2 + 2ab + b^2 = (a + b)^2$$

- **Square of a Binomial – Subtraction**

$$a^2 - 2ab + b^2 = (a - b)^2$$

- **Difference of Two Squares**

$$a^2 - b^2 = (a + b)(a - b)$$

- **Difference of Two Cubes**

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

- **Sum of Two Cubes**

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Factoring Methods

- **GCF Method**

Sometimes, the terms of a polynomial all share some common factors. The product of these common factors is called the greatest common factor (GCF) of all the terms.

- **Grouping Method**

When a polynomial has four or more terms, it is sometimes necessary to use the grouping method to factor. Group terms in a polynomial such that each group shares a common factor. Then factor the GCF from each group.

- **Factoring With a Leading Coefficient of 1**

To factor a trinomial of the form $x^2 + px + q$, find two numbers, a and b, such that: $a + b = p$ and $ab = q$. Then use the following identity to factor the trinomial:

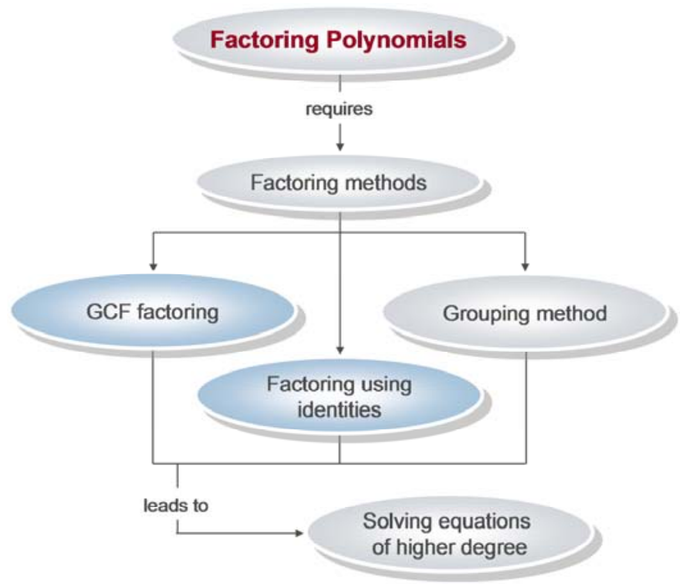
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

- **Factoring With a Leading Coefficient of *A***

To factoring a trinomial of the form $Ax^2 + Bx + C$:

1. Find the product of A and C.
2. Find two numbers, p and q, with a sum of B and a product of C.
3. Factor A into two numbers, u and v, such that p and q are each divisible by one of the factors.
4. Assume that p is divisible by u, and q is divisible by v. Let $m = p/u$ and $n = q/v$.
5. The factored form of the trinomial will be: $(ux + n)(vx + m)$

Concept Map



Example: Solve By Factoring

Solve the equation by factoring:

$$x^2 + 3x - 28 = 0$$

Solution:

Using a table, find the two numbers with a sum of 3 and a product of -28.

$a = -28$ $b = 1$	$a = 28$ $b = -1$	$a = -14$ $b = 2$	$a = 14$ $b = -2$	$a = -7$ $b = 4$	$a = 7$ $b = -4$
$p = -27$ $q = -28$	$p = 27$ $q = -28$	$p = -12$ $q = -28$	$p = 12$ $q = -28$	$p = -3$ $q = -28$	$p = 3$ $q = -28$

These numbers are 7 and -4, so:

$$(x + 7)(x - 4) = 0$$

Set each factor equal to 0 and solve:

$$x + 7 = 0 \qquad x - 4 = 0$$

$$x = -7 \qquad x = 4$$

Tips & Reminder

- Factoring is finding polynomials whose product is a given polynomial.
- Factorable polynomials can be converted to the product of some other polynomials.
- Prime polynomials cannot be converted to the product of at least two factors other than 1.
- If all the terms in a polynomial share a common factor, then the first step in the factoring process is to factor out the GCF.
- When a polynomial has four or more terms, it may be necessary to us the grouping method to factor.
- Some special polynomials require using identities backwards to factor.
- Some equations of higher degree can be solved using factoring if they are of the form polynomial = 0.

How to Use This Cheat Sheet: These are the key concepts related this topic. Try to read through it carefully twice then rewrite it out on a blank sheet of paper. Review it again before the exam.