Rapid Learning Center

Chemistry :: Biology :: Physics :: Math

Rapid Learning Center Presents …

Teach Yourself CLEP College Algebra Visually in 24 Hours

*CLEP is a registered trademark of the College Board, which does not endorse, nor is affiliated in any way with the Rapid Learning courses.

1/40

Factoring Polynomials

CLEP College Algebra Rapid Learning Series

Wayne Huang, Ph.D.
Susan Kim, Ph.D.
Mark Cowan, Ph.D.
Diop El Moctar, Ph.D.
Poornima Gowda, Ph.D.
Daniel Deaconu, Ph.D.
Fabio Mainardi, Ph.D.
Theresa Johnson, M.Ed.
Jessica Davis, M.S.
Wendy Perry, M.A.

Rapid Learning Center

www.RapidLearningCenter.com/

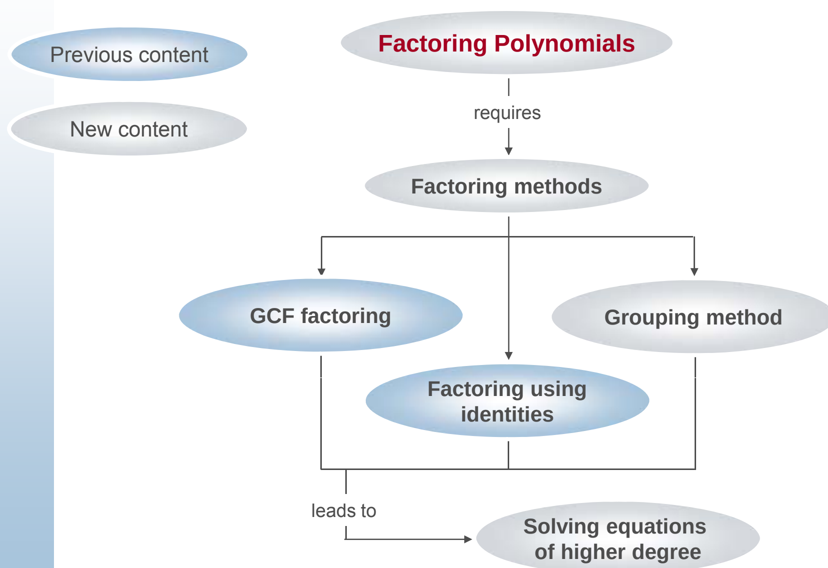
Learning Objectives

By completing this tutorial, you will learn how to:

- Distinguish between prime and factorable polynomials
- Factor using the GCF method
- Factor using the grouping method
- Apply identities to factor polynomials
- Solve equations using factoring

3/40

Concept Map



4/40

Factoring Polynomials

Definition: Factoring

Factoring – The process of finding polynomials whose product is equivalent to a given polynomial.

6/40

Example: Factoring

The binomials $(x - 3)$ and $(x + 4)$ are two factors whose product is

$$x^2 + x - 12$$

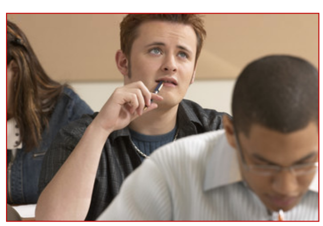
Therefore, this trinomial can be factored as the product $(x - 3)(x + 4)$.

7/40

Definition: Factorable and Prime

Factorable Polynomial – A polynomial that can be converted to the product of some other polynomials.

Prime Polynomial – A polynomial that cannot be converted to the product of at least two factors other than 1.



8/40

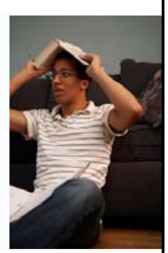
Examples: Factorable and Prime

Examples of factorable and prime polynomials include:

Prime Polynomial	$x^2 + x + 1$
Factorable Polynomial	$x^3 - 27$
Factorable Polynomial	$x^2 + 8x + 16$
Prime Polynomial	$x + 1$

9/40

GCF & Grouping Methods



Greatest Common Factor

Sometimes, the terms of a polynomial all share some common factors.

The product of these common factors is called the **greatest common factor** (GCF) of all the terms.

11/40

Note: First Step in Factoring

If all the terms in a polynomial share a common factor, then the **first step** in the factoring process is to **factor out the GCF**.

12/40

Example: Greatest Common Factor

Factor out the GCF of the polynomial:

$$21m^4n^3 - 7m^5n^3 + 49m^7n^7$$

Solution:

$$21m^4n^3 - 7m^5n^3 + 49m^7n^7$$
$$= 7(3m^4n^3 - m^5n^3 + 7m^7n^7)$$
$$= 7m^4(3n^3 - mn^3 + 7m^3n^7)$$
$$= 7m^4n^3(3 - m + 7m^3n^4)$$

13/40

Grouping Method

When a polynomial has four or more terms, it is sometimes necessary to use the **grouping method** to factor.

Group terms in a polynomial such that each group shares a common factor. Then factor the GCF from each group.

14/40

Example: Grouping Method

Factor the polynomial by grouping:

$xy - 2nx + 3y - 6n$

Solution:

$xy - 2nx + 3y - 6n$

$= (xy - 2nx) + (3y - 6n)$

$= x(y - 2n) + 3(y - 2n)$

$= \mathbf{(y - 2n)(x + 3)}$

15/40

Special Factoring Identities

Special Factoring Identities

Identities are used to factor some special polynomials.

In this method, use identities backwards to find the prime factors of special polynomials.

17/40

Square of a Binomial – Addition

Square of a Binomial – Addition

$$a^2 + 2ab + b^2 = (a + b)^2$$

When a trinomial includes the sum of the squares of two terms plus twice the product of these terms, factor the trinomial as the square of the sum of the terms.

18/40

Example 1: Square of a Binomial

Factor the polynomial:

$$25u^2 + 10uv + v^2$$

Solution:

$25u^2 + 10uv + v^2$

$= (5u)^2 + 2(5u)(v) + v^2$

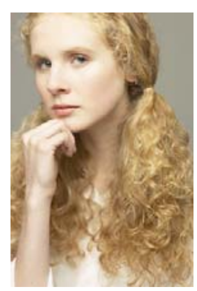
$= (5u + v)^2$

19/40

Square of a Binomial – Subtraction

Square of a Binomial – Subtraction

$$a^2 - 2ab + b^2 = (a - b)^2$$



When a trinomial includes the sum of the squares of two terms minus twice the product of these terms, factor the trinomial as the square of the difference of the terms.

20/40

Example 2: Square of a Binomial

Factor the polynomial:

$$4x^2 - 12x + 9$$

Solution:

$4x^2 - 12x + 9$

$= (2x)^2 - 2(2x)(3) + 3^2$

$= (2x - 3)^2$

21/40

Difference of Two Squares

Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b)$$

When a binomial is made up of the difference of two squared terms, factor the binomial as the product of the sum and difference of the terms.

22/40

Example: Difference of Two Squares

Factor the polynomial:

$$81a^4 - b^6$$

Solution:

$$81a^4 - b^6 = (9a^2)^2 - (b^3)^2$$

$$= (9a^2 + b^3)(9a^2 - b^3)$$

23/40

Difference of Two Cubes

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

When a binomial is made up of the difference of two cubed terms, factor the binomial as the product of the difference of the terms and a trinomial formed by the sum of the squares of the terms plus the product of the terms.

24/40

Example: Difference of Two Cubes

Factor the polynomial:

$$x^6 - 125$$

Solution:

$x^6 - 125$

$= (x^2)^3 - (5)^3$

$= (x^2 - 5)\,[(x^2)^2 + (x^2)(5) + 5^2]$

$= (x^2 - 5)(x^4 + 5x^2 + 25)$

25/40

Sum of Two Cubes

Sum of Two Cubes

$$a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$$

When a binomial is made up of the sum of two cubed terms, factor the binomial as the product of the sum of the terms and a trinomial formed by the sum of the squares of the terms minus the product of the terms.

26/40

Example: Sum of Two Cubes

Factor the polynomial:

$$27m^3 + 64n^3$$

Solution:

$27m^3 + 64n^3$

$= (3m)^3 + (4n)^3$

$= (3m + 4n)\,[(3m)^2 - (3m)(4n) + (4n)^2]$

$= \mathbf{(3m + 4n)(9m^2 - 12mn + 16n^2)}$

27/40

Factoring Trinomials

Factoring With a Leading Coefficient of 1

To factor a trinomial of the form

$$x^2 + px + q$$

find two numbers, *a* and *b*, such that:

$$a + b = p \qquad ab = q$$

Then use the following identity to factor the trinomial:

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

29/40

Example 1: Leading Coefficient of 1

Factor the polynomial:

$$x^2 + x - 12$$

Solution:

$a = -12$, $b = 1$	$a = 12$, $b = -1$	$a = -6$, $b = 2$	$a = 6$, $b = -2$	$a = -4$, $b = 3$	$a = 4$, $b = -3$
$p = -11$	$p = 11$	$p = -4$	$p = 4$	$p = -1$	$p = 1$
$q = -12$	$q = -12$	$q = -12$	$q = -12$	$q = -12$	$q = -12$

$$x^2 + x - 12 = (x + 4)(x - 3)$$

30/40

Example 2: Leading Coefficient of 1

Factor the polynomial:

$$x^2 - 9x + 20$$

Solution:

$a = 4$, $b = 5$	$a = -4$, $b = -5$	$a = -10$, $b = -2$	$a = 10$, $b = 2$	$a = 20$, $b = 1$	$a = -20$, $b = -1$
$p = 9$	$p = -9$	$p = -12$	$p = 12$	$p = 21$	$p = -21$
$q = 20$	$q = 20$	$q = 20$	$q = 20$	$q = 20$	$q = 20$

$$x^2 - 9x + 20 = (x - 4)(x - 5)$$

31/40

Factoring With a Leading Coefficient of *A*

To factor a trinomial of the form $Ax^2 + Bx + C$, do the following:

1. Find the product of *A* and *C*.
2. Find two numbers, *p* and *q*, with a sum of *B* and a product of *AC*.
3. Factor *A* into two numbers, *u* and *v*, such that *p* and *q* are each divisible by one of the factors.
4. Assume that *p* is divisible by *u*, and *q* is divisible by *v*. Let $m = p/u$ and $n = q/v$.
5. The factored form of the trinomial will be

$$(ux + n)(vx + m)$$

32/40

Example: Leading Coefficient of *A* (pt. 1)

Factor the polynomial:

$6x^2 + 13x + 5$

Solution:

$(5)(6) = 30 \quad \rightarrow \quad p = 10, q = 3$

$u = 2, v = 3$

33/40

Example: Leading Coefficient of *A* (pt. 2)

Factor the polynomial:

$6x^2 + 13x + 5$

Solution:

$(5)(6) = 30 \quad \rightarrow \quad p = 10, q = 3$

$u = 2, v = 3$

$\rightarrow \quad m = p/u = 10/2 = 5$

$n = q/v = 3/3 = 1$

$6x^2 + 13x + 5 = \mathbf{(2x + 1)(3x + 5)}$

34/40

Solving Equations

Solving Equations Using Factoring

Some equations of higher degree can be solved using factoring.

The equation must be of the form

polynomial = 0

36/40

Example: Solve By Factoring

Solve the equation by factoring:

$$x^2 - 9x + 14 = 0$$

Solution:

$$x^2 - 9x + 14 = 0$$

$$(x - 2)(x - 7) = 0$$

$x - 2 = 0$ $\qquad$ $x - 7 = 0$

$\mathbf{x = 2}$ $\qquad$ $\mathbf{x = 7}$

37/40

Learning Summary

Factoring is finding polynomials whose product is a given polynomial.

Prime polynomials cannot be converted to the product of at least two factors other than 1.

Some **special polynomials** require using identities backwards to factor.

Factorable polynomials can be converted to the product of some other polynomials.

Some equations of **higher degree** can be solved using factoring.

38/40

Congratulations

You have successfully completed the core tutorial

Factoring Polynomials

Rapid Learning Center

Rapid Learning Center

Chemistry :: Biology :: Physics :: Math

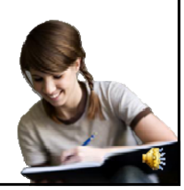
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!



40/40

http://www.RapidLearningCenter.com