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Teach Yourself CLEP College Algebra Visually in 24 Hours

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Quadratic Equations

CLEP College Algebra Rapid Learning Series

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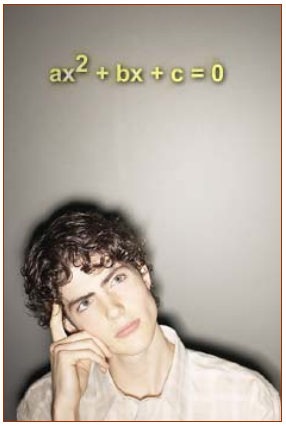
Learning Objectives

By completing this core tutorial, you will:



- **Define quadratic functions.**
- **Identify the graphs of quadratic functions along with their properties.**
- **Solve quadratic equations by factoring, the square root method, completing the square and the quadratic formula.**
- **Use imaginary numbers in quadratic equation solutions.**
- **Identify and solve quadratic inequalities.**

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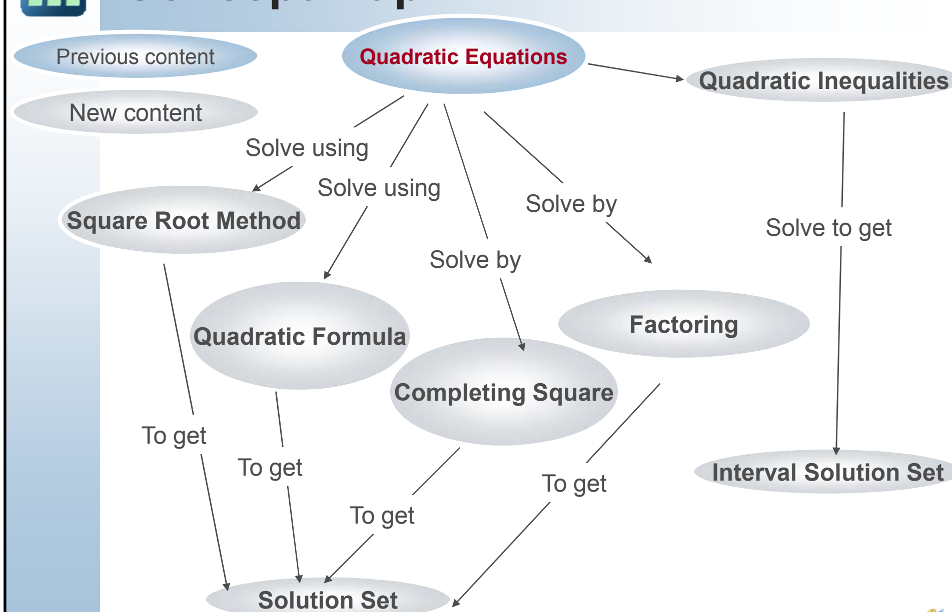
Concept Map



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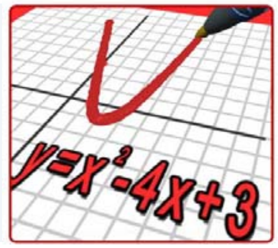
Quadratic Functions Defined



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Definition – Quadratic Function

A **quadratic function** is a specific type of function written in the standard form:

$$f(\boldsymbol{x}) = a\boldsymbol{x}^2 + b\boldsymbol{x} + c = 0, a \neq 0$$

Quadratic term — linear term — constant term

If $a = 0$, there would be no quadratic term.

Examples:

$$2x^2 - 2x + 1 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 1 = 0$$

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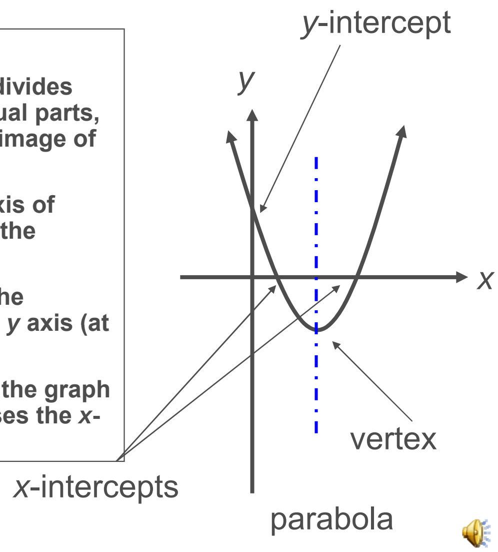
Graph – Quadratic Function

The graph of a quadratic function is a parabola.

- **Graph Parts**
  - **Axis of Symmetry** - divides parabola into two equal parts, each part is a mirror image of the other.
  - **Vertex** – where the axis of symmetry intersects the parabola.
  - ***Y*-intercept** – where the parabola crosses the y axis (at $x = 0$).
  - ***X*-intercepts** – where the graph of the parabola crosses the x-axis (at $y = 0$)



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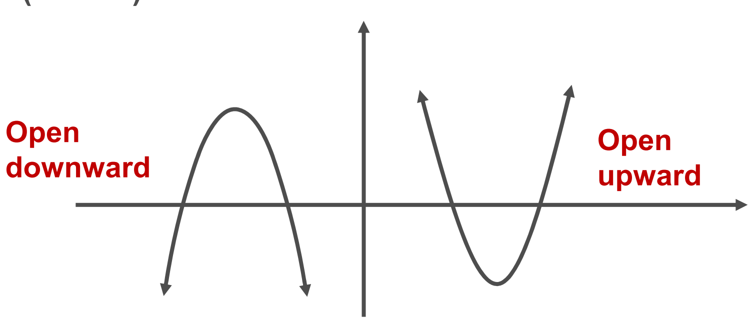
Parabola Properties – Open Up or Down

Parabolas may open upward (if $a > 0$) or downward (if $a < 0$).



Tip: Use your hands to help you remember the shape of a parabola.

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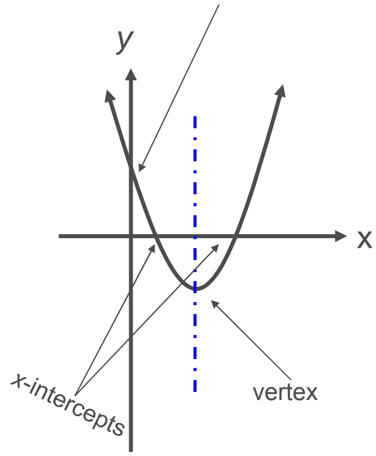
The Mathematics of the Parabola

Given below are the formulas used to find the parts of the parabola.

- **Axis of Symmetry:** $x = -\frac{b}{2a}$
- **Vertex**: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
- ***Y*-intercept: (0, c)**
- ***X*-intercepts:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



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Maximum and Minimum

The vertex of a parabola can be a maximum or minimum value.

If $a > 0$, the vertex is a minimum.

If $a < 0$, the vertex is a maximum.

Examples:

$$3x^2 + 9x + 6 = 0$$

- $a > 0$, this vertex is a minimum (3 > 0).

$$-5x^2 + 6x + 1 = 0$$

- $a < 0$, this vertex is a maximum (-5 < 0).

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Solving Quadratic Equations

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Methods of Solving Quadratic Equations

There are four methods of solving quadratic equations.

- Factoring
- Square Root Method
- Completing the Square
- Using the Quadratic Formula

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Factor an Expression

To factor an expression means to rewrite it as a product.

<u>Example</u>

Factor: $2x^2 - x$

$$2x^2 - x = x(2x - 1)$$

In this problem, we used the Greatest Common Factor (GCF), *x*.

The GCF is the largest factor that elements in an expression have in common.

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Zero Product Property

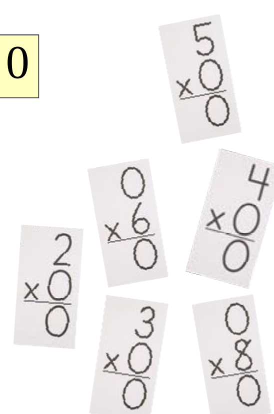
The Zero Product Property states that, if:

$$ab = 0, \text{ then } a = 0 \text{ or } b = 0$$

<u>Example</u>

If $(x + 4)(x - 2) = 0$,

then $(x + 4) = 0$ or $(x - 2) = 0$



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Solve by Factoring

If the left side of an equation can be factored, then the zero-product property can be used to solve the quadratic equation.

Example: Solve $x^2 + 2x = 15$ by factoring.

Rewrite equation in standard form: $x^2 + 2x - 15 = 0$

Factor the left side: $(x+5)(x-3) = 0$

Apply zero-product rule: $x + 5 = 0$ or $x - 3 = 0$

Solve for x in each equation: $x = -5$ or $x = 3$

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Square Root Method

The square root method states that if A and B are algebraic expressions such that $A^2 = B$, then …

$$A = \sqrt{B} \quad \text{or} \quad A = -\sqrt{B}.$$

We can solve a quadratic equation by the square root method if we can write it in the form $A^2 = B$.

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Solving by the Square Root Method

Example: Solve equation $(3x-1)^2-9=0.$

Apply square root method: $(3x-1)^2 = 9$

$$3x-1=\sqrt{9} \text{ or } 3x-1=-\sqrt{9}$$

$$3x-1=3 \text{ or } 3x-1=-3$$

Solve equations: $3x-1+1=3+1 \text{ or } 3x-1+1=-3+1$

$$3x=4 \text{ or } 3x=-2$$

$$x=\frac{4}{3} \text{ or } x=-\frac{2}{3}$$

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Definition: Perfect Square Trinomial

Perfect Square Trinomial – A product obtained when a binomial is squared.

Example

$x^2 - 4x + 4$ can be written as $(x - 2)^2$

Properties of Perfect Square Trinomials:

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

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The Difference of Two Squares

The difference of two squares can be written as:

$$a^2 - b^2 = (a - b)(a + b)$$

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Completing the Square

<u>Completing the square</u> involves finding the last term of a perfect square trinomial to solve an equation.

Formula: $$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

<u>Example</u>

Find the value to complete the square: $x^2 + 6x +$ __.

Calculate $(b/2)^2$: $(6/2)^2 = 3^2 = 9$

This value completes the square: $x^2 + 6x + \underline{\mathbf{9}} = (x + 3)^2$

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Steps to Completing the Square

Completing the Square:

1. Find $(b/2)^2$
2. Rewrite equation so that all x terms are on one side.
3. Complete the square by adding $(b/2)^2$ to both sides.
4. Factor the perfect square trinomial.
5. Apply the square root method.
6. Solve for x.

$x^2 - 10x + 13 = 0$

1. $(-10/2)^2 = 25$
2. $x^2 - 10x = -13$
3. $x^2 - 10x + 25 = -13 + 25$
4. $(x - 5)^2 = 12$
5. $(x - 5) = \sqrt{12}$
6. $x - 5 = -\sqrt{12}$ or $x - 5 = \sqrt{12}$

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Recall the Quadratic Formula

The quadratic formula helps us to find solutions to quadratic equations of the form $ax^2 + bx + c = 0$.

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: Be sure to plug in the values of a, b, and c exactly as they are. Alterations can lead to strange, incorrect answers.

Example: Find the solutions of the quadratic equation

$x^2 - 8x - 9.$

1. Identify the coefficients of the given quadratic equation.

$x^2 - 8x - 9$ → a, b, c

a = 1, b = - 8, c = - 9

2. Plug the coefficients into the formula.

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-9)}}{2(1)}$$

3. Simplify the formula.

$$\frac{8 \pm \sqrt{100}}{2} = \frac{8 \pm 10}{2}$$

4. Find solutions.

$$\frac{8+10}{2} = \frac{18}{2} = 9 \qquad \frac{8-10}{2} = \frac{-2}{2} = -1$$

The solutions are 9 and -1

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Solutions to Quadratic Equations

The number and type of solutions of a quadratic equation depends on the sign of the discriminant, $b^2 - 4ac$.

- If $b^2 - 4ac > 0$, then the equation has two distinct real solutions:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

x_1 and x_2 are the roots of the quadratic equation

- If $b^2 - 4ac = 0$, then the equation has one real solution:

$$x = -\frac{b}{2a}$$

- If $b^2 - 4ac < 0$, then the equation has two imaginary solutions (no real solutions) : What are imaginary solutions?

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Complex or Imaginary Numbers

Imaginary numbers – Numbers of the form *a* + *bi*, where *a* and *b* are real numbers (b ≠ 0).

- ***bi* indicates the part of the expression that is imaginary.**
- **$i^2 = -1$**

Example: $\sqrt{-9} = i\sqrt{9} = i \cdot 3 = 3i$

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Solving with the Quadratic Formula

Example: Solve equation $2x^2 - x - 3 = 0$.

- Identify *a*, *b* and *c* $a = 2,\ b = -1$ and $c = -3$
- Evaluate $b^2 - 4ac$ $(-1)^2 - 4 \times 2 \times (-3) = 25$
- Plug *a*, *b* and *c* in quadratic formula

$$x = \frac{-(-1) + \sqrt{(-1)^2 - 4(2)(-3)}}{2 \times 2} \text{ or } x = \frac{-(-1) - \sqrt{(-1)^2 - 4(2)(-3)}}{2 \times 2}$$

$$x = \frac{1 + \sqrt{25}}{4} = \frac{6}{4} \text{ or } x = \frac{1 - \sqrt{25}}{4} = -\frac{4}{4}$$

- Solution $x = \frac{3}{2}$ or $x = -1$

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Stop-and-Think

1) **True or false** : Quadratic equation $ax^2 + bx + c = 0$ has two distinct roots if $a^2 - 4bc > 0$.

False. The discriminant is $b^2 - 4ac$.

2) **True or false** : Quadratic equation $ax^2 + bx + c = 0$ has two distinct roots if $b^2 - 4ac > 0$.

True.

3) **True or false** : If $b^2 - 4ac = 0$ then Quadratic equation $ax^2 + bx + c = 0$ has no roots.

False. If $b^2 - 4ac = 0$, there is one root or solution.

4) **True or false** : $2x + 1 = 0$ is a quadratic equation.

False. This equation has no quadratic term.

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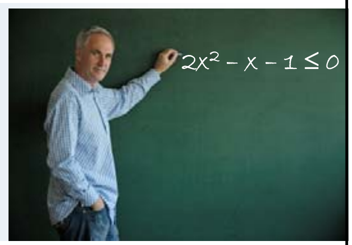
Quadratic Inequalities



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Definition- Quadratic Inequality

Quadratic Inequality – A quadratic equation where the equal sign is replaced by an inequality symbol.

Examples: $2x^2 - x + 1 \geq 0$

$x^2 - 2x + 1 \leq 0$

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Solving a Quadratic Inequality

Solving a quadratic inequality:

1. Put the inequality in standard form.
2. Define boundaries.
3. Denote the test interval.
4. Find the sign (-, +) in every interval.
5. Determine which interval satisfies the quadratic inequality.

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Quadratic Inequality Example - 1

Solve the following Quadratic Inequality: $x^2 + 3x < 18$

1. Put the equation in standard form.

   $x^2 + 3x - 18 < 0$

2. Define boundaries.
   - To define boundaries, change the inequality to an equality, $x^2 + 3x - 18 = 0$
   - Find the solution of the equation, $(x + 6)(x - 3) = 0$
   - $x = -6$ or 3
3. Denote the test intervals.

   The test intervals are $(-\infty, -6)$, $(-6, 3)$, $(3, \infty)$.

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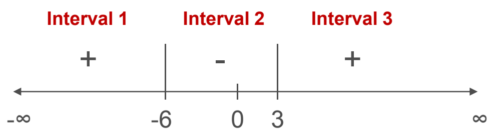
Quadratic Inequality Example - 2

Solve the following Quadratic Inequality: $x^2 + 3x - 18 < 0$

4 **Find the sign (-, +) in every interval.**

To find the sign in each interval, use a test value from the interval (plug this value in to the quadratic equation).



$f(-7) = (-7)^2 + 3(-7) - 18$

$= 10$ (positive)

$f(1) = (1)^2 + 3(1) - 18$

$= -14$ (negative)

$f(5) = (5)^2 + 3(5) - 18$

$= 22$ (positive)

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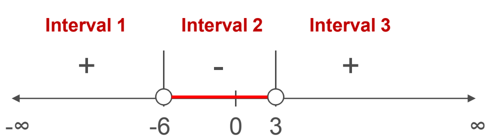
Quadratic Inequality Example - 3

Solve the following Quadratic Inequality: $x^2 + 3x - 18 < 0$

5 **Determine which interval satisfies the quadratic inequality.**



In our case, the quadratic inequality should be less than zero (< 0).

The only interval satisfying this condition is interval 2, (-6, 3).

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Learning Summary

The quadratic function graph has an axis of symmetry, vertex, x-intercepts and y-intercepts.

You can solve a quadratic equation by factoring, the square root method, completing the square, or the quadratic formula.

A quadratic function is a specific type of function written in the standard form: $ax^2 + bx + c = 0$

A quadratic Inequality is a quadratic equation where the equal sign is replaced by an inequality sign.

The solution to a quadratic inequality is an interval set.

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Congratulations

You have successfully completed the core tutorial

Quadratic Equations

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What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!

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