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Teach Yourself

CLEP College Mathematics Visually in 24 Hours

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Logic

CLEP College Mathematics Rapid Learning Series

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Learning Objectives

After completing this tutorial, you will be able to:

- Define logic
- Determine the truth value of a conditional statement
- Recognize variations of a conditional statement
- Define Boolean Algebra

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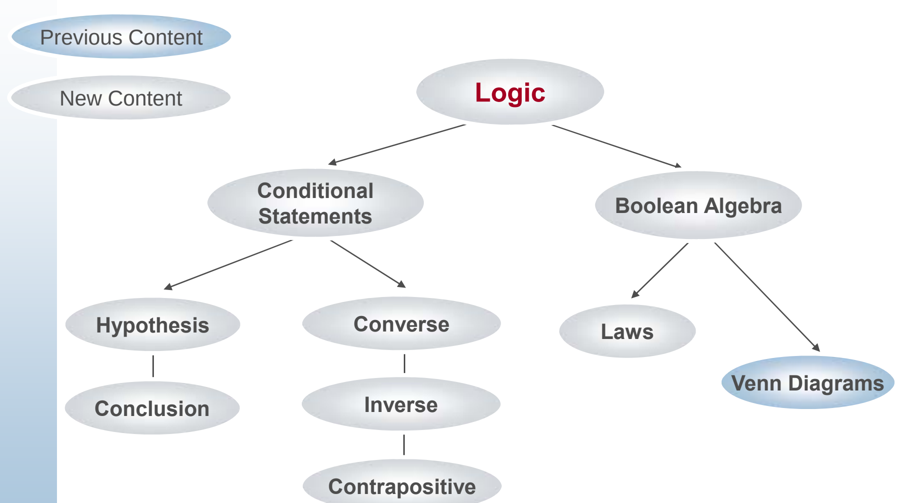
Concept Map



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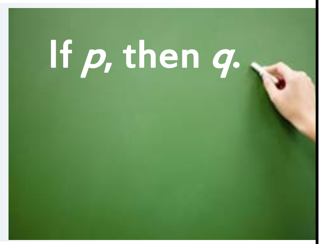
Conditional Statements



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Definition: Logic

Logic – The science that investigates the rules controlling reliable inference.

This branch of science involves special vocabulary, notation, and methodology for determining what is true and false.

Logic is used to construct formal proofs in all branches of mathematics.

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Definition: Conditional Statement

Conditional statement: a statement that can be put in the form "if *p*, then *q*."

Examples:

1. If Larry gets 18 questions correct on the final exam, then his final grade will be a B.
2. If it is a leap year, then there are 366 days in the year.
3. If it is hot outside, then the consumption of ice cream increases.

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Parts of a Conditional

A conditional statement has two parts.

- The part that follows "if" is the **hypothesis**.
- The part that follows "then" is the **conclusion**.

Use $\Rightarrow$ to represent a conditional statement.

This symbol also translates to "implies".

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Example: Hypothesis and Conclusion

Identify the hypothesis and conclusion:

If it is a weekday, then the office is open.

Solution:

Hypothesis → It is a weekday.

Conclusion → The office is open.

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Definition: Truth Value

Truth value – whether a conditional statement is true or false.

Examples:

1) If a polygon has three sides, then it is a triangle.
   - Truth value → true

2) If there are two people in front of Danny in a line, then Danny is fourth in line.
   - Truth value → false

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Definition: Counterexample

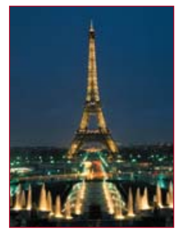
Counterexample – used to prove a conditional statement is false.

The counterexample will show the hypothesis is true and the conclusion is false.

Example:

Statement → If you are in Paris, then you are in France.

Counterexample → The city of Paris, Illinois is in the Unites States, not France.



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Definition: Necessary Condition

If we say that "*p* is a **necessary condition** for *q*", we mean that if we don't have *p*, then we won't have *q*.

Or put differently, without *p*, you won't have *q*.

To say that *p* is a necessary condition for *q* does not mean that *p* guarantees *q*.

Example: Air is a necessary condition for human life.

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Definition: Sufficient Condition

If we say that "*p* is a **sufficient condition** for *q*", then we mean that if we have *p*, we know that *q* must follow.

In other words, *p* guarantees *q*.

Example: A sufficient condition for getting from home to work is having a chauffeured limo ride.

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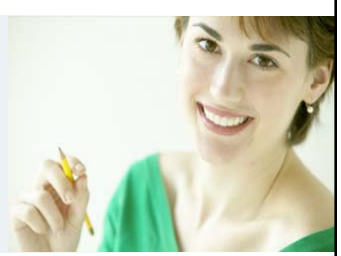
Variations on Conditional Statements



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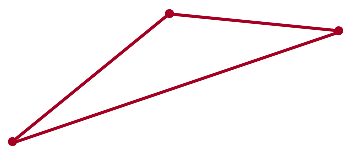
Definition: Converse

The **converse** of the conditional statement "if *p*, then *q*" is the conditional "if *q*, then *p*."

Example:

Statement → If a polygon has three sides, then it is a triangle.

Converse → If a polygon is a triangle, then it has three sides.



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Truth Value and Converse

The **truth value** of the converse of a conditional statement is not always the same as the original statement.

Example:

Statement → If an animal is a turtle, then it is a reptile.

- Truth value → true

Converse → If an animal is a reptile, then it is a turtle.

- Truth value → false

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Example: Converse

Write the converse of this statement.

If you are in the city of Los Angeles, then you are in the state of California.

Solution:

Converse → If you are in the state of California, then you are in the city of Los Angeles.

Truth value → false

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Definition: Negation

Negation – The opposite meaning of the original statement.

By definition, the negation of a conditional statement has the opposite truth value.

Use ~ (a tilde) to represent the negation of a conditional statement.

Example:

Statement → The sky is blue.

Negation → The sky is not blue.

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Definition: Inverse

The **inverse** of the conditional statement "if *p*, then *q*" is the conditional "if not *p*, then not *q*."

Find the inverse of a conditional statement by **replacing the hypothesis and conclusion with their negations**.

There is no set rule for the truth value of an inverse. Some conditional statements may have the same truth value as their inverse, others may not.

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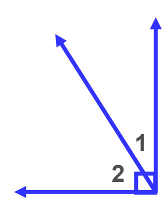
Example: Inverse

Write the inverse of this statement.

If the sum of the measures of two angles is 90°, then the two angles are complements.



Solution:

If the sum of the measures of two angles is not 90°, then the two angles are not complements.

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Definition: Contrapositive

The **contrapositive** of the conditional statement "if *p*, then *q*" is the conditional "if not *q*, then not *p*."

A conditional statement and its contrapositive always have the same truth value.

Example: If a number is a whole number, then it is an integer.

Contrapositive → If a number is not an integer, then it is not a whole number.

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Example: Contrapositive

Write the contrapositive of this statement.

If the sum of the measures of two angles is 90°, then the two angles are complements.

Solution:

If two angles are not complements, then the sum of the measures of the two angles is not 90°.

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Example: Conditional Statement

Choose the inverse of this statement.

If a point has negative coordinates, then it is in the third quadrant.

A. If a point is in the third quadrant, then it has negative coordinates.

B. If a point does not have negative coordinates, then it is not in the third quadrant.

C. If a point is not in the third quadrant, then it does not have negative coordinates.

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Definition: Conjunction

Conjunction – Two statements combined with the word "and".

Use $\wedge$ (the upward carrot symbol) to represent a conjunction.

A conjunction is only true if both of the original statements are true. Otherwise, the conjunction is false.

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Definition: Disjunction

Disjunction – Two statements combined with the word "or".

Use ∨ (the downward carrot symbol) to represent a disjunction.

A disjunction is true if at least one of the original statements is true.

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Example: Conjunction vs. Disjunction

Which statement is a conjunction?

A. The leaves are green.

B. It is springtime.

C. The leaves are green or it is springtime.

D. The leaves are green and it is springtime.

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Definition: Truth Table

A **truth table** shows the truth values of the hypothesis, conclusion, and some of the related statements.

Example:

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

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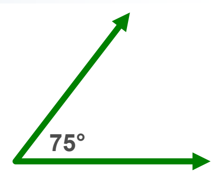
Example: Truth Table

If an angle has a positive measure less than 90°, then it is an acute angle.

Determine if the statement is true for the diagram.



Solution:

Truth value → true

75° → positive

$75° < 90°$

75° is an acute angle.

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Boolean Algebra

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Definition: Boolean Algebra

Boolean algebra is the algebra of logic. It is a symbolic representation of statements that are either true or false.

There are three basic **Boolean operators**:

- And, represented by $\wedge$
- Or, represented by $\vee$
- Not, represented by ~

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Laws of Boolean Algebra

Some laws of Boolean algebra are:

Associative	$a \vee (b \vee c) = (a \vee b) \vee c$
	$a \wedge (b \wedge c) = (a \wedge b) \wedge c$
Commutative	$a \vee b = b \vee a$
	$a \wedge b = b \wedge a$
Distributive	$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
	$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

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More Laws of Boolean Algebra

Some additional laws of Boolean algebra include:

Identity	$a \vee a = a$
	$a \wedge a = a$
Complements	$a \vee {\sim}a = 1$
	$a \wedge {\sim}a = 0$
De Morgan's Law	${\sim}(a \vee b) = {\sim}a \wedge {\sim}b$
	${\sim}(a \wedge b) = {\sim}a \vee {\sim}b$

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Example: Boolean Algebra

Simplify the expression.

$$(A \vee B) \wedge \sim(C \vee D) \vee (A \vee B) \wedge (C \vee D)$$

Solution:

$$(A \vee B) \wedge \sim(C \vee D) \vee (A \vee B) \wedge (C \vee D)$$

$$(A \vee B) \wedge [\sim(C \vee D) \vee (C \vee D)]$$

$$(A \vee B) \wedge 1$$

$$A \vee B$$

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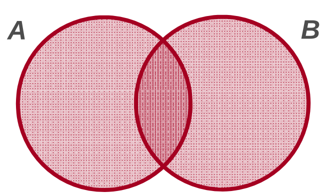
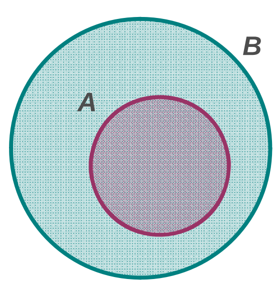
Definition: Venn Diagram

A **Venn diagram** is a visual organizer consisting of two or more overlapping circles. The circles represent similarities and differences between concrete concepts.





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Constructing a Venn Diagram

To construct a Venn diagram:

1. Identify the concepts being compared.
2. List the similarities and differences of the concepts.
3. Draw and label a circle for each concept with appropriate overlapping for similarities.
4. List the common characteristics of concepts in the overlapping areas; list unique characteristics in the non-overlapping areas.

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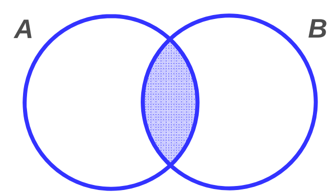
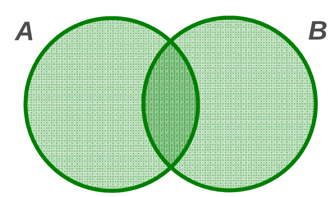
Venn Diagrams and Boolean Algebra

In a Venn diagram, the **intersection** of sets *A* and *B* is the overlap of the two sets.



The **union** of the sets, *A* or *B*, includes everything in both sets.



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Example: Venn Diagrams

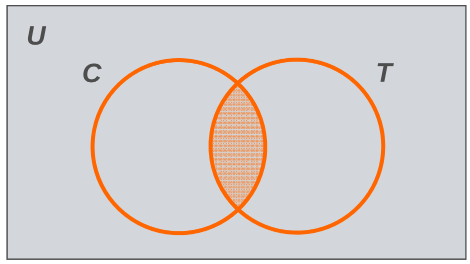
Use a Venn diagram to represent the statement.

Some cars have only two doors.

U = all vehicles

C = all cars

T = all two-door vehicles



The **intersection** of *C* and *T* represents the given statement.

$C \wedge T$

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Learning Summary

Variations of conditional statements include the **converse**, **inverse**, and **contrapositive**.

Logic is the study of the rules governing inference.

Boolean algebra is the algebra of logic.

A conditional statement contains a **hypothesis** and a **conclusion**.

A **conditional statement** can be written in the form "if *p*, then *q*."

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Congratulations

You have successfully completed
the core tutorial

Logic

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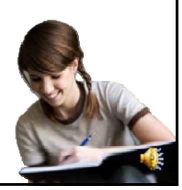
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!



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