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Teach Yourself CLEP Precalculus Visually in 24 Hours

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Basic Trigonometry

CLEP Precalculus Rapid Learning Series

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Learning Objectives

By completing this tutorial, you will learn concepts of:

- The unit circle
- Right triangles
- The six trigonometric functions
- Periodic behavior of trigonometric functions
- Fundamental identities of trigonometric functions
- Common values of trigonometric functions
- The use of reference angles

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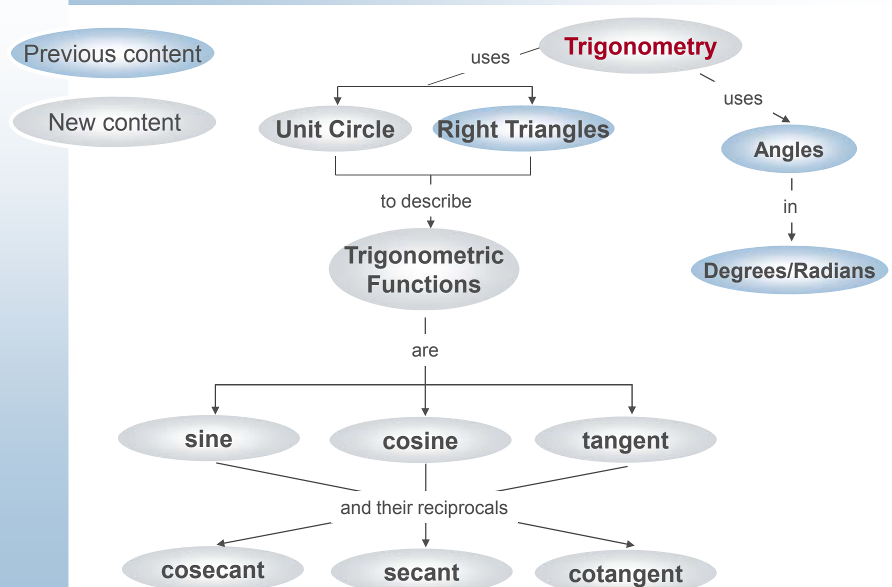
Concept Map



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Unit Circle and Trigonometric Functions

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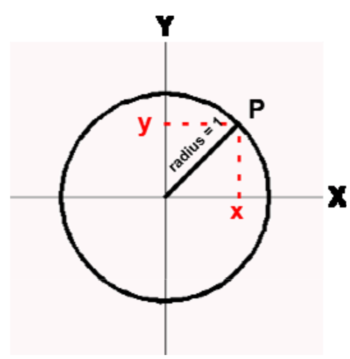
Definition: Unit Circle

Unit circle – The circle of radius one centered at the origin.

- Let (x, y) be the coordinates of a point P on a unit circle.
- The coordinates (x, y) obey the equation:

$$x^2 + y^2 = 1$$



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Example: Unit Circle

Does the point ($^3/_5$, $^4/_5$) lie on the unit circle?

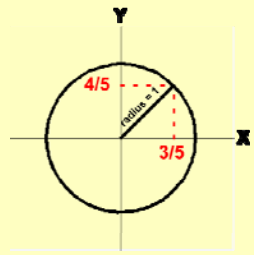
Solution:

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{3^2}{5^2} + \frac{4^2}{5^2} = \frac{9}{25} + \frac{16}{25}$$

$$= \frac{9+16}{25}$$

$$= \frac{25}{25}$$

$$= 1$$



Yes, ($^3/_5$, $^4/_5$) lies on the unit circle.

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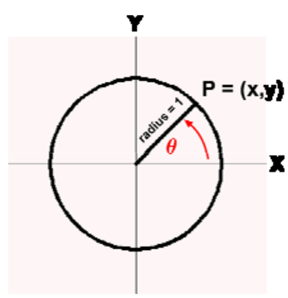
Unit Circle and Trigonometric Functions

The six trigonometric functions are:

$\sin\theta = y$	$\csc\theta = 1/y$ where $y \neq 0$
$\cos\theta = x$	$\sec\theta = 1/x$ where $x \neq 0$
$\tan\theta = y/x$ where $x \neq 0$	$\cot\theta = x/y$ where $y \neq 0$



Note: $\sin\theta = {}^1/_{\csc\theta}$, $\cos\theta = {}^1/_{\sec\theta}$, and $\tan\theta = {}^1/_{\cot\theta}$.

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Trigonometric Functions in Unit Circle

The point (3/5, 4/5) lies on the unit circle. Find the six trigonometric functions for this point.

Solution:

$\sin\theta = y = 4/5$ $\csc\theta = 1/y = 5/4$

$\cos\theta = x = 3/5$ $\sec\theta = 1/x = 5/3$

$\tan\theta = y/x = 4/3$ $\cot\theta = x/y = 3/4$

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Definitions: Domain and Range

Domain – The set of possible values of *θ*; input.

Range – The set of possible values of *f(θ)*; output.

Function	Domain	Range
sin θ	all real numbers	[-1, 1]
cos θ	all real numbers	[-1, 1]
tan θ	$\theta \neq \pi/2 + k\pi$	all real numbers
csc θ	$\theta \neq k\pi$	(-∞, -1] and [1, ∞)
sec θ	$\theta \neq \pi/2 + k\pi$	(-∞, -1] and [1, ∞)
cot θ	$\theta \neq k\pi$	all real numbers

**k* = any integer

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Example: Domain and Range

Which value does not belong to the domain of tan θ?

- [ ] π
- [ ] 2π
- [x] 5π/2
- [ ] 6π/2
- [ ] 8π/2

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Definition: Periodic Function

Periodic Function – A function that repeats itself at regular intervals.

Period – Any interval over which a function repeats itself.

Fundamental Period – The smallest interval over which a periodic function repeats itself.

Examples: The six trigonometric functions are examples of periodic functions.

■ 360° (2π radians) is the fundamental period of sin θ, cos θ, csc θ, and sec θ.

■ 180° (π radians) is the fundamental period of tan θ and cot θ.

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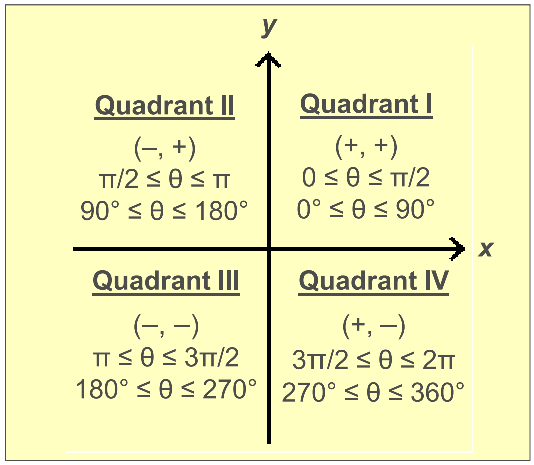
Definition: Quadrants

The coordinate plane is divided by its axes into four quadrants.



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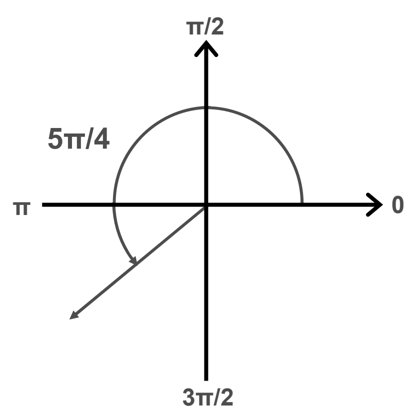
Example: Quadrants

In which quadrant does the standard angle 5π/4 radians lie?

- [ ] I
- [ ] II
- [x] III
- [ ] IV



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Signs of Trigonometric Functions

The quadrant a standard angle lies in tells the sign of each trigonometric function.

*"**A**ll **S**tudents **T**ake **C**alculus"*

	Quadrant II (left of y-axis)	Quadrant I (right of y-axis)
Above x-axis	**Students** **sin θ > 0** cos θ < 0 tan θ < 0	**All** **sin θ > 0** **cos θ > 0** **tan θ > 0**
Below x-axis	sin θ < 0 cos θ < 0 **tan θ > 0** **Take**	sin θ < 0 **cos θ > 0** tan θ < 0 **Calculus**

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Example: Trigonometric Function Signs

In which quadrant is the product (cos θ · sin θ) negative?

Solution: A product of two numbers is negative if the numbers have opposite signs.

Quadrants where sine and cosine have opposite signs: II and IV

Where product (cos θ · sin θ) is negative:
Quadrants II and IV

	Left of y-axis	Right of y-axis
Above x-axis	**Students** **sin θ > 0** cos θ < 0 tan θ < 0	**All** **sin θ > 0** **cos θ > 0** **tan θ > 0**
Below x-axis	**Take** sin θ < 0 cos θ < 0 **tan θ > 0**	**Calculus** sin θ < 0 **cos θ > 0** tan θ < 0

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Right Triangles

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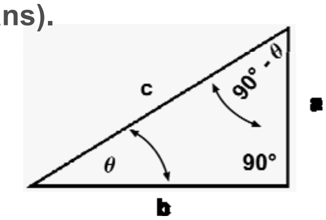
Definition: Right Triangle

Right triangle – A geometric figure having three sides and three angles with exactly one angle with a measure of 90° (π/2 radians).



- The sum of the three angles of any triangle is always 180° (π radians).
- The sum of the two non-right angles in a right triangle is 90° (π/2 radians).
- Given θ, the other non-right angle is 90° – θ (π/2 – θ).

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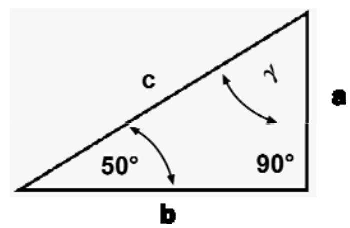
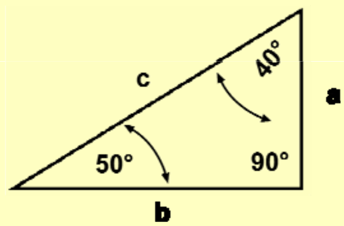
Example: Missing Angle

Find the measure of γ in a triangle with given angle measures 50° and 90°.



Solution: The sum of the two non-right angles must be equal to 90°.

$$\gamma + 50° = 90°$$
$$\gamma = 90° - 50°$$
$$\gamma = 40°$$



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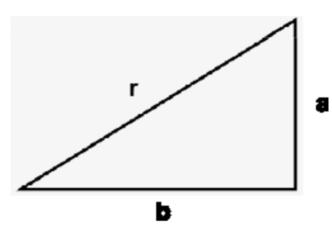
Definition: Pythagorean Theorem

Pythagorean Theorem

The square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs of a right triangle.

$$r^2 = a^2 + b^2$$

- **The hypotenuse is the longest side of a right triangle; opposite the right angle.**
- **The legs of a right triangle are the two shortest sides; join to form the right angle.**



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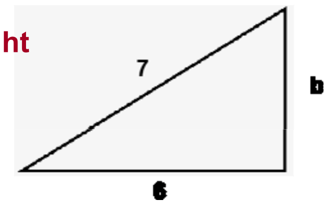
Example: Pythagorean Theorem

What is the length of leg *b* of a right triangle where the hypotenuse length is 7 and the other leg is 6?



Solution: Use the Pythagorean theorem to find *b*.

$$r^2 = a^2 + b^2$$
$$7^2 = 6^2 + b^2$$
$$b^2 + 6^2 = 7^2$$
$$b^2 + 36 = 49$$

$$b^2 = 49 - 36$$
$$b^2 = 13$$
$$\mathbf{b = \sqrt{13}}$$

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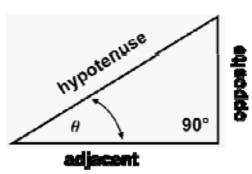
Right Triangle and Trig Functions

The six trigonometric functions can be defined as ratios of the lengths of the sides of a right triangle.



"SOHCAHTOA"

$$\text{Sin}\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{Csc}\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\text{Cos}\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{Sec}\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\text{Tan}\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \text{Cot}\theta = \frac{\text{adjacent}}{\text{opposite}}$$

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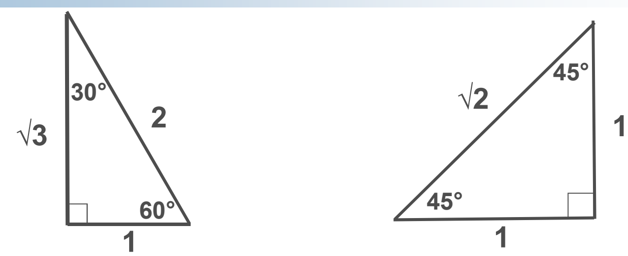
Common Angles of Trigonometry



Angle θ	sin θ	cos θ	tan θ
30° (or π/6)	1/2	√3/2	√3/3
45° (or π/4)	√2/2	√2/2	1
60° (or π/3)	√3/2	1/2	√3

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Reciprocal and Quotient Identities

The following are the reciprocal identities:

sin θ = 1/csc θ   csc θ = 1/sin θ

cos θ = 1/sec θ   sec θ = 1/cos θ

tan θ = 1/cot θ   cot θ = 1/tan θ

The following are the quotient identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

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Pythagorean and Cofunction Identities

The following are the Pythagorean identities:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

The following are the cofunction identities:

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\csc(90° - \theta) = \sec\theta$

$\sec(90° - \theta) = \csc\theta$

$\cot(90° - \theta) = \tan\theta$

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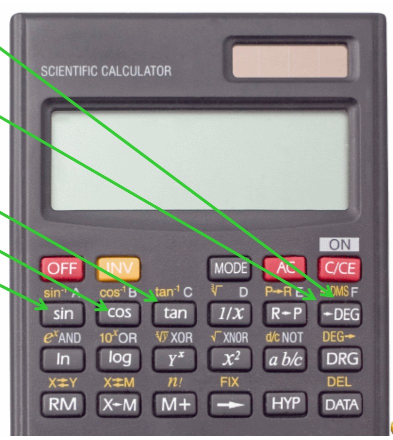
Standard Keys

Scientific calculators have the ability to change between degrees and radians.

Scientific calculators also have keys for the following trigonometric functions:

- tan
- cos
- sin



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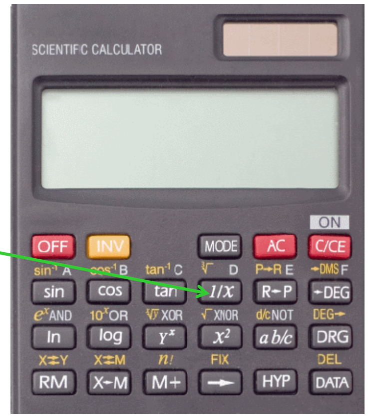
Calculator: Trig Function Keys

Some calculators may not have keys for cosecant, secant, and cotangent.

Their values can be calculated by using the key corresponding to the appropriate reciprocal function (cos, sin, or tan) in combination with the reciprocal key.



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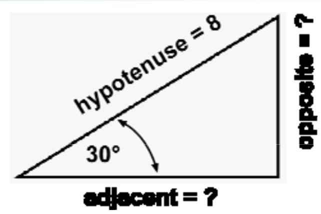
Example: Application

Find the length of the opposite side of the triangle.



Solution:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30^\circ = \frac{\text{opposite}}{8}$$

$$\text{opposite} = 8 \cdot \sin 30^\circ$$

$$\text{opposite} = 8 \cdot 0.5$$

opposite = 4

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Reference Angles

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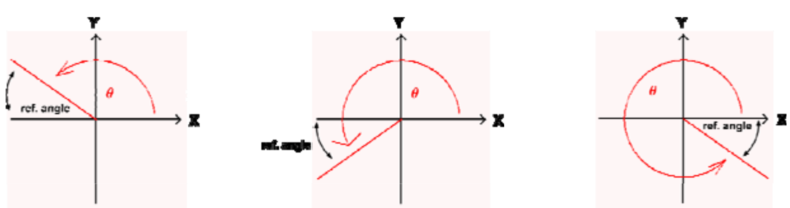
Definition: Reference Angle

Reference angle – Given angle θ, the acute angle between the terminal side of θ and the x-axis.



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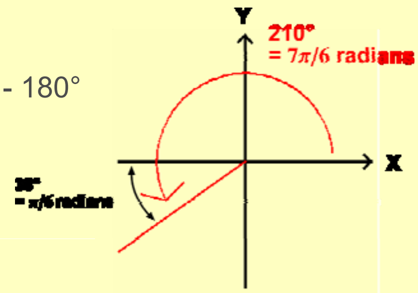
Example: Reference Angle

Find the reference angle of 210° (or 7π/6 radians).

Solution:

reference angle = 210° - 180°

= 30°



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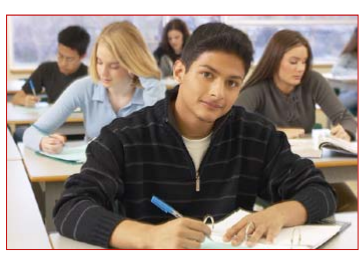
Evaluating Obtuse Angles

We can evaluate trigonometric functions of obtuse angles using the following steps:

Step 1: Find the trigonometric function value of the reference angle corresponding to the obtuse angle θ.

Step 2: Determine the sign of the function value using the trigonometric sign rules (All Students Take Calculus).



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Example: Obtuse Angles

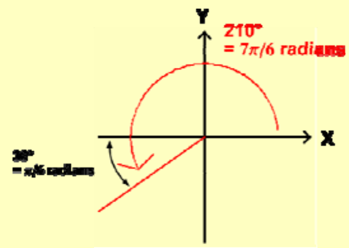
Evaluate tan 210°.

Solution:

reference angle = 30°

tan 30° = √3/3

tan 210° = √3/3



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Learning Summary

"**All Students Take Calculus**" can help you recall the sign rules.

On the **unit circle** the point (x, y) is cos θ and sin θ, respectively.

The trigonometric functions are **periodic**.

Use **SOHCAHTOA** to recall the ratios that define the trigonometric functions.

The **inverse** of sine, cosine, and tangent, are cosecant, secant, and cotangent, respectively.

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Congratulations

You have successfully completed
the core tutorial

Basic Trigonometry

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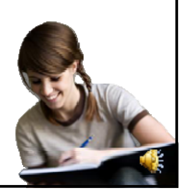
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)
→ **Step 2: Practice – Interactive Problem Drill**
Step 3: Recap – Super Review Cheat Sheet

Go for it!



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