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Teach Yourself Trigonometry Visually in 24 Hours

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Graphs of Trigonometric Functions

Trigonometry Rapid Learning Series

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Learning Objectives

By completing this tutorial, you will learn concepts of:

- The basic graphs of the trigonometric functions
- The general form of the sine and cosine curves
- Amplitudes and periods of trigonometric functions
- Transformations

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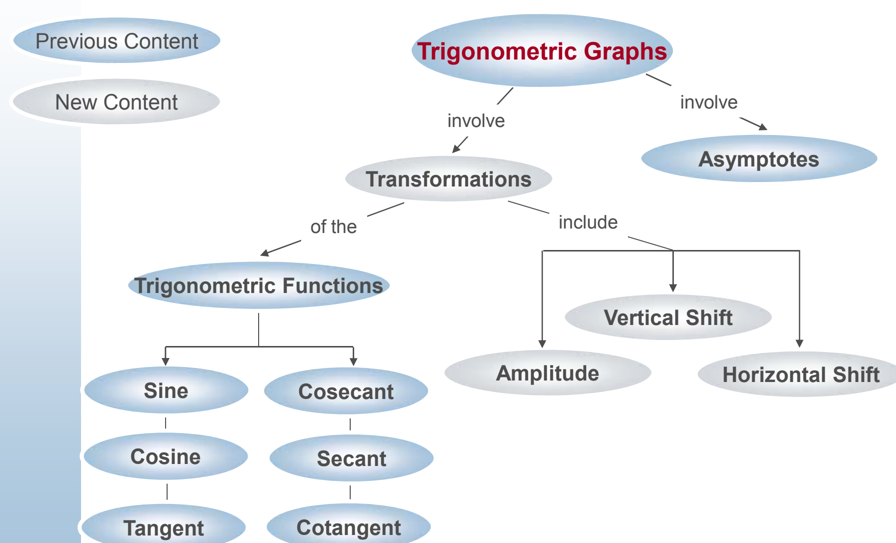
Concept Map



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Sine and Cosine Graphs

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Definition: Key Points

Key points can be used to plot the graph of a function.

- **In drawing the graph of a function, choose key points that correspond to the:**
  - **maximum values of a function**
  - **minimum values of a function**
  - ***x*-intercepts of a function (an *x*-intercept is a point at which the value of a function is equal to zero)**

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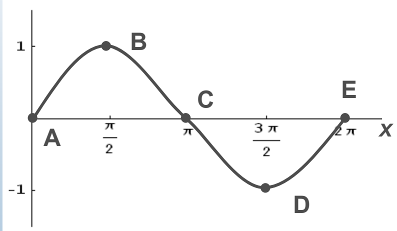
Graph: $y = \sin x$

Use five key points to plot the graph of $y = \sin x$.

$y = \sin x$



Point	x	y	Type

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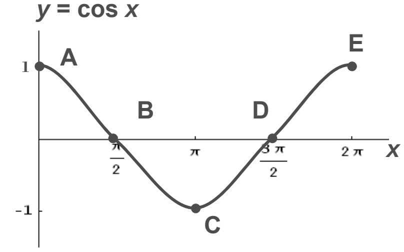
Graph: y = cos x

Use five key points to plot the graph of $y = \cos x$.

Point	x	y	Type

$y = \cos x$



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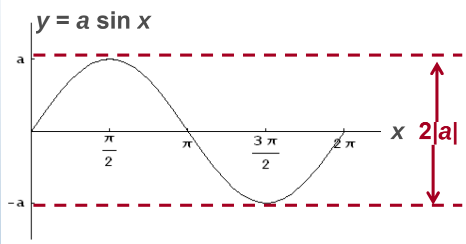
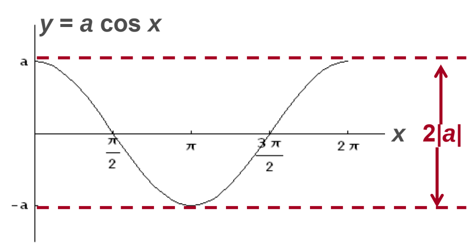
Definition: Amplitude

Amplitude – One-half of the vertical distance between the maxima and minima of a sine or cosine graph.





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Identifying the Amplitude

The general forms of the sine and cosine functions are:

$y = d + a\sin(bx - c)$ $\qquad$ $y = d + a\cos(bx - c)$

The absolute value of "*a*" is the **amplitude** of the function.

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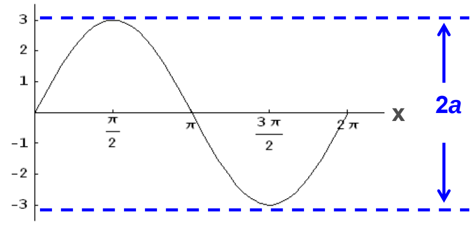
Example: Amplitude

Find the equation for the sine graph shown.



Solution:

Vertical distance between the max and min:

$$3 - (-3) = 3 + 3 = 6$$

Amplitude: $|a| = 6/2 = 3$

Equation: $y = 3 \sin x$

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Identifying the Period

Fundamental Period – The smallest interval over which a periodic function repeats itself.

$y = d + a \sin(bx - c)$ $\qquad$ $y = d + a \cos(bx - c)$

The constant "*b*" is directly related to the period of sine and cosine.

$$\text{period} = \frac{2\pi}{b}$$

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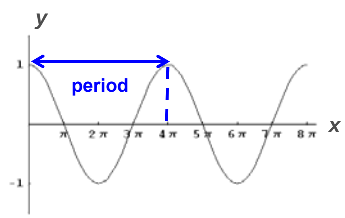
Example: Period

Find the equation for the cosine graph shown.



Solution:

period = 4π	$b = 2\pi/4\pi$
period = $2\pi/b$	$b = ½$
$4\pi = 2\pi/b$	**Equation: $y = \cos(x/2)$**

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Transformations

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Transformations - Outline

Transformations on the trigonometric functions include:

- Vertical translations
- Horizontal translations
- Changes in amplitude and period
- Phase shifts
- Exponential damping

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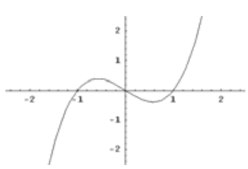
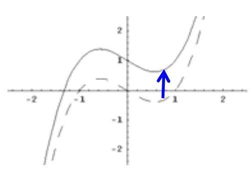
Vertical Translation

Vertical Translation – An upward or downward shift in the graph of a function.

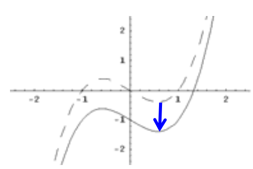
Function



Upward Shift



Downward Shift



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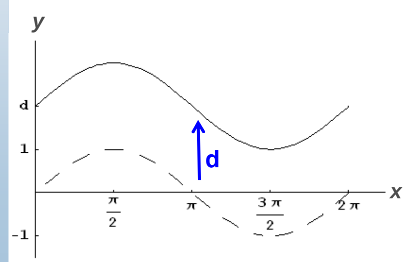
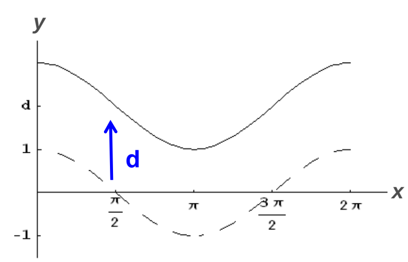
Identifying Vertical Translations

y = *d* + *a* sin(*bx* – *c*)   *y* = *d* + *a* cos(*bx* – *c*)

The constant "*d*" determines the magnitude and direction of the vertical shift of a sine or cosine graph.





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Example: Vertical Translation

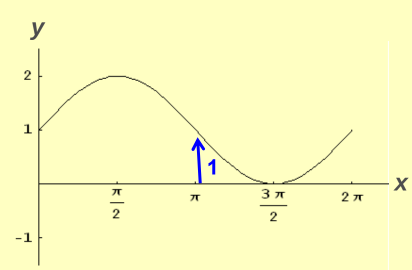
Plot the graph of *y* = 1 + sin *x*.

Solution:

We begin by plotting *y* = sin *x*.

Since *d* = 1, shift *y* = sin *x* upward by 1 unit.

y = 1 + sin *x* →



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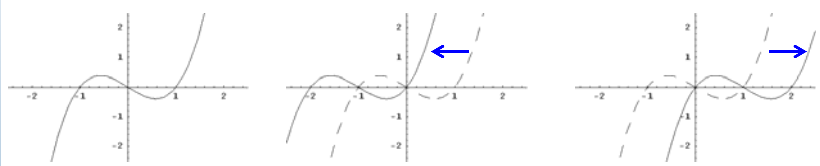
Horizontal Translation

Horizontal Translation – A shift to the left or right of the graph of a function.

Function	Shift to the Left	Shift to the Right



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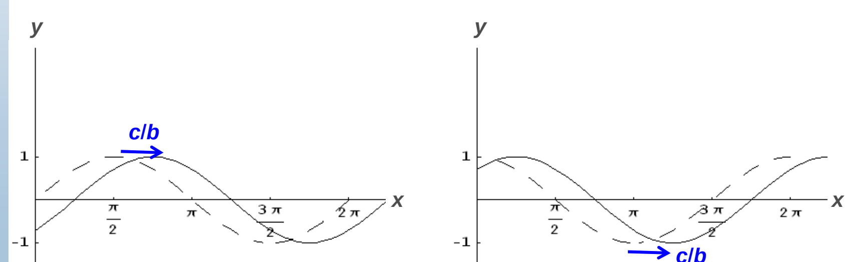
Identifying Horizontal Translations

$y = d + a\sin(bx - c)$ $y = d + a\cos(bx - c)$

The ratio c/b is called the **phase shift**. The ratio c/b determines the magnitude and direction of the horizontal translation.



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Example: Horizontal Translation

Plot the graph of $y = \cos(x + \pi/5)$.

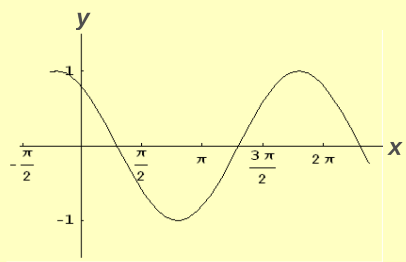
Solution:

$y = \cos(x + \pi/5)$

$y = \cos(x - (-\pi/5))$

$b = 1$, $c = -\pi/5$

$c/b = -\pi/5$



Since c/b is negative, shift $y = \cos x$ to the left by $\pi/5$ units.

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Example: Multiple Transformations

Plot the graph of $y = 1 + 2\cos(2x + \pi/3)$.

Solution:

Amplitude: $|a| = 2$

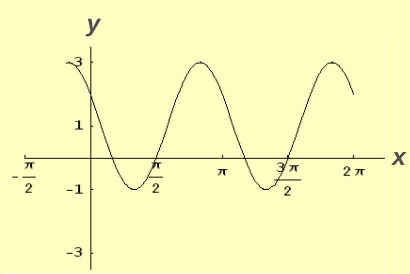
Period:

$2\pi/b = 2\pi/2 = \pi$

Horizontal Shift:

$c/b = (-\pi/3)/2 = -\pi/6$

Vertical Shift: $d = 1$



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Example: Exponential Decay

Draw the graph of $y = 3e^{-0.1x} \sin x$.

Solution:

$y = a \sin bx$

$a = 3e^{-0.1x}$ $b = 1$

Amplitude:

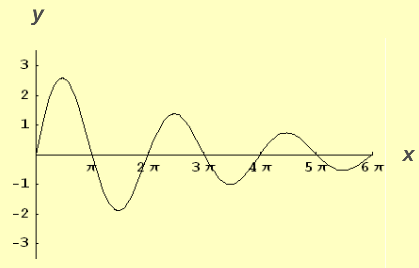
$|a| = 3e^{-0.1x}$

- plot $3e^{-0.1x}$ and $-3e^{-0.1x}$

Period:

$2\pi/b = 2\pi/1 = 2\pi$

*Remove the exponential curve.



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Tangent and Cotangent Graphs

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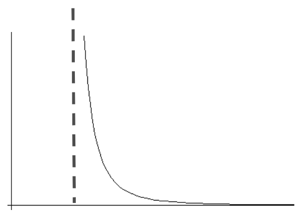
Definition: Asymptote

Asymptote – Any line that a function approaches closely without ever intersecting.

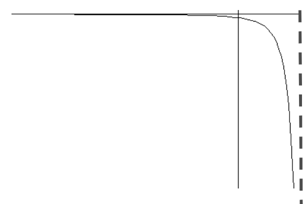
Vertical asymptote – A vertical line that a function approaches closely without ever intersecting.

As a function approaches a vertical asymptote it will either become:



increasingly positive

or



increasingly negative

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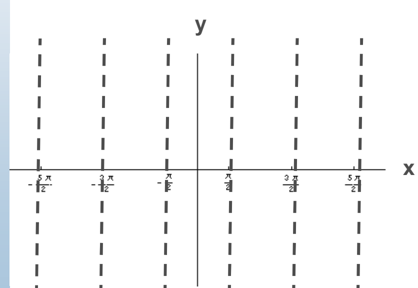
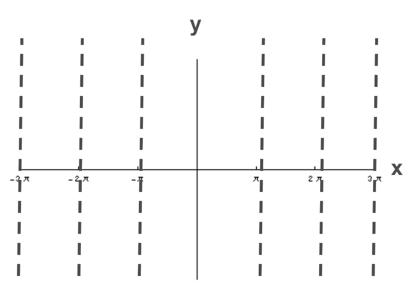
Asymptotes of Tangent and Cotangent

The function $y = \tan x$ has a vertical asymptote at each **odd integer multiple of π/2** (…, -3π/2, -π/2, π/2, 3π/2, …).

The function $y = \cot x$ has a vertical asymptote at each **multiple of π** (…, -3π, -2π, -π, π, 2π, 3π,…).



Asymptotes of Tangent



Asymptotes of Cotangent

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How-to: Graph *y* = tan *x*

To graph *y* = tan *x*:

1. Draw a pair of consecutive vertical asymptotes.
2. Tabulate and plot 7 key points.
   - 3 key points will be located between the left asymptote and the midpoint
   - 1 key point will be the midpoint between the asymptotes
   - 3 key points will be located between the midpoint and the right asymptote

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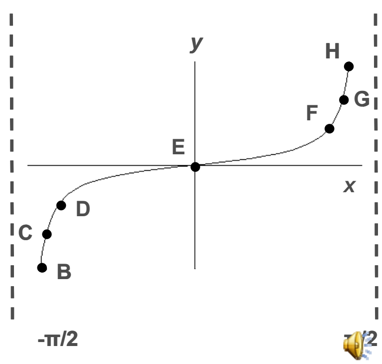
Graph: *y* = tan *x*

Use key points to plot the graph of *y* = tan *x*.

	x	*y*	Type



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Graph of $y = a \cdot \tan(bx)$

Consider the function $y = a \cdot \tan(bx)$.

- **The period of this function will be π/b.**
- **If the constant "*a*" is positive, the graph of a period will be increasing. (Figure 1)**
- **If the constant "*a*" is negative, the graph of a period will be decreasing. (Figure 2)**

Figure 1

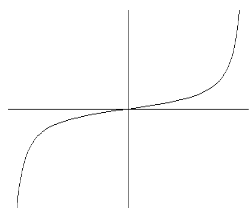
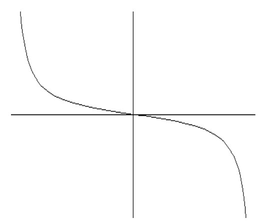


Figure 2



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Example: Tangent Transformation

Draw the graph of $y = 2 \tan(x/2)$.

Solution:

$a = 2$ $b = ½$

Period:

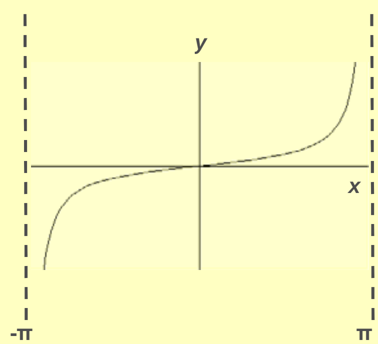
$\pi/b = \pi/(1/2)$
$= 2\pi$

Consecutive asymptotes:

$x = -\pi$ and π

Behavior:

$a > 0 \rightarrow$ increasing interval $(-\pi, \pi)$



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How-to: Graph *y* = cot *x*

To graph *y* = cot *x*:

1. Draw a pair of consecutive vertical asymptotes.
2. Tabulate and plot 7 key points.
   - 3 key points will be located between the left asymptote and the midpoint
   - 1 key point will be the midpoint between the asymptotes
   - 3 key points will be located between the midpoint and the right asymptote

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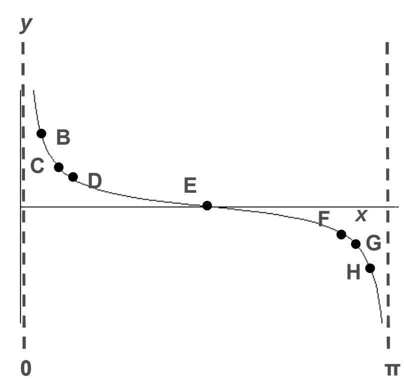
Graph: *y* = cot *x*

Use key points to plot the graph of *y* = cot *x*.

	x	*y*	Type



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Graph of $y = a \cot(bx)$

Consider the function $y = a \cdot \cot(bx)$.

- **The period will be π/b.**
- **If the constant "*a*" is positive, the graph of a period will be decreasing.**
- **If the constant "*a*" is negative, the graph of a period will be increasing.**

Figure 1

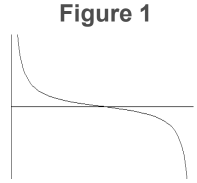
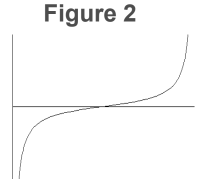


Figure 2



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Example: Cotangent Transformation

Draw the graph of $y = -3 \cot(x/6)$.

Solution:

$a = -3 \qquad b = 1/6$

Period:

$\pi/b = \pi/(1/6)$

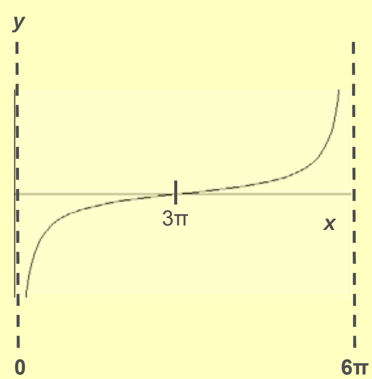
$= 6\pi$

Consecutive asymptotes:

$x = 0$ and 6π

Behavior:

$a < 0 \rightarrow$ increasing interval $(0, 6\pi)$



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Secant and Cosecant Graphs

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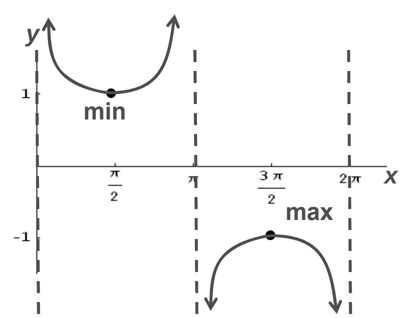
Graph: $y = \csc x$

Use five key points to plot the graph of $y = \csc x$.

- Use the fact that $\csc x$ is the reciprocal of $\sin x$.

x	$\sin x$	$y = \csc x$



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Example: Cosecant Transformation

Draw the graph of $y = \csc(2x - \pi/3)$.

Solution:

$y = \csc(bx - c)$

$b = 2$ $c = \pi/3$

Period:

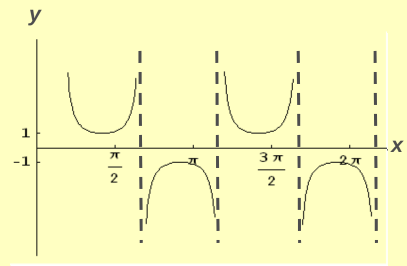
$2\pi/b = 2\pi/2 = \pi$

Horizontal shift:

$c/b = (\pi/3)/2 = \pi/6$

Vertical asymptotes:

$x = 2\pi/3, 7\pi/6, 5\pi/3, 13\pi/6$



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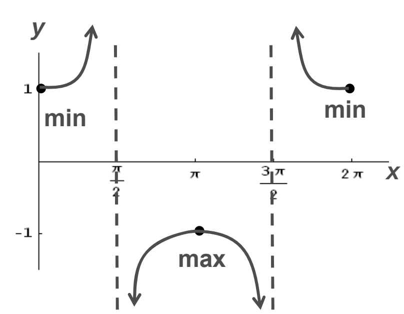
Graph: *y* = sec *x*

Use five key points to plot the graph of $y = \sec x$.

- **Use the fact that sec *x* is the reciprocal of cos *x*.**

x	cos *x*	*y* = sec *x*



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Example: Secant Transformation

Draw the graph of $y = -1 + 2 \sec x$.

Solution:

$y = d + a \cdot \sec x$

$a = 2 \qquad d = -1$

Amplitude:

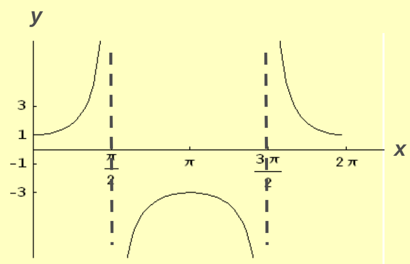
$|a| = |2| = 2$

Vertical shift:

$d = -1 \rightarrow$ down 1 unit

Vertical asymptotes:

$x = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$



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Learning Summary

The **period** of a sine or cosine function is $2\pi/b$.

The **vertical shift** of a trigonometric function is determined by the value of d.

The **period** of a tangent or cotangent function is π/b.

The **amplitude** is determined by $|a|$ in the general form equations of sine and cosine.

The **horizontal shift** of a trigonometric function is determined by the ratio c/b.

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Congratulations

You have successfully completed
the core tutorial

Graphs of Trigonometric Functions

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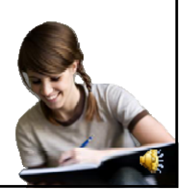
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!



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