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Precalculus in 24 Hours

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The Unit Circle and Angle Measure

Precalculus Rapid Learning Series

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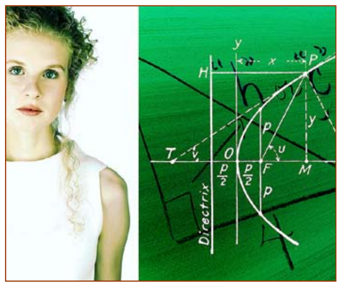
Learning Objectives

By completing this tutorial, you will learn concepts of:



- The unit circle
- Radian and degree angle measures
- Positive and negative angles
- Complementary and supplementary angles
- Arc length and area of sectors

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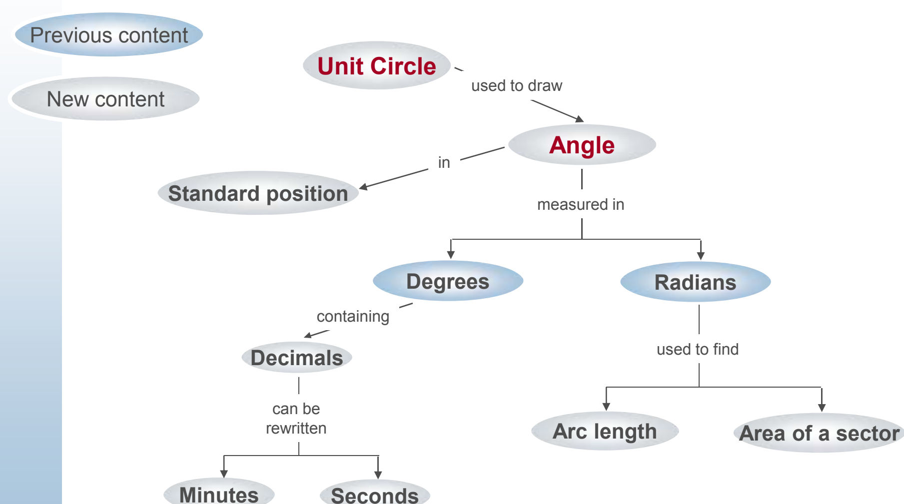
Concept Map



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Unit Circle

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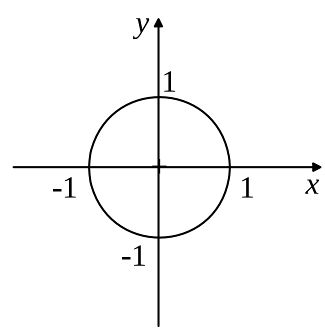
Definition: Unit Circle

Unit circle – n. a circle centered at the origin of the coordinate plane with radius 1.

Equation: $x^2 + y^2 = 1$

Circumference: 2π units

Area: π square units



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Properties of the Unit Circle

Some properties of the unit circle are:

- The unit circle is symmetric with respect to the *x*-axis, the *y*-axis, and the origin.
- The ratio of the circumference of any circle with radius *r* to the circumference of the unit circle is *r*.
- The ratio of the area of any circle with radius *r* to the area of the unit circle is r^2.

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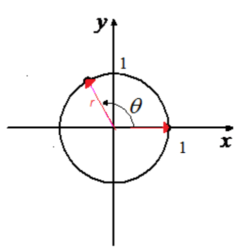
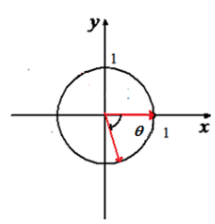
Definition: Standard Position

Standard position – when an angle θ is drawn with its initial side on the positive *x*-axis and its terminal side on a radius *r* of a circle.

Angle θ is **positive** if measured in the counterclockwise direction.

Angle θ is **negative** if measured in the clockwise direction.





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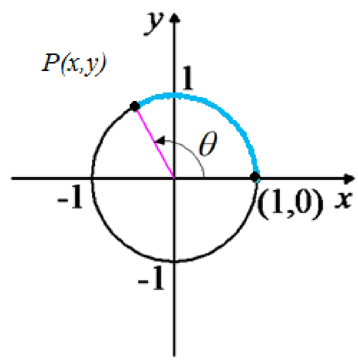
Circular Arcs on the Unit Circle

By rotating a ray through an angle θ in standard position, a **circular arc** is formed along the circumference from the point (1, 0) to the point *P*(*x*, *y*).



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Angles

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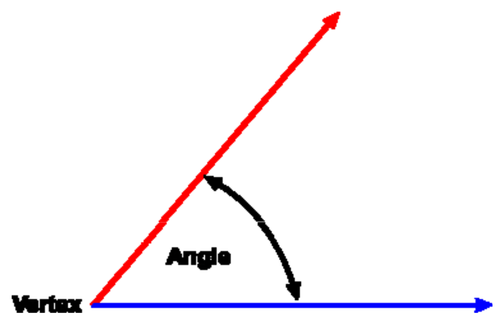
Definition: Angle

Angle – n. two rays extending from a single point called the vertex.



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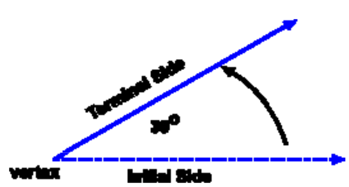
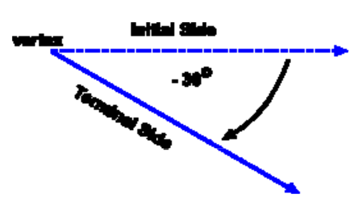
Positive and Negative Angles

A positive angle:
formed by rotating a ray in the counterclockwise direction.

A negative angle:
formed by rotating a ray in the clockwise direction.





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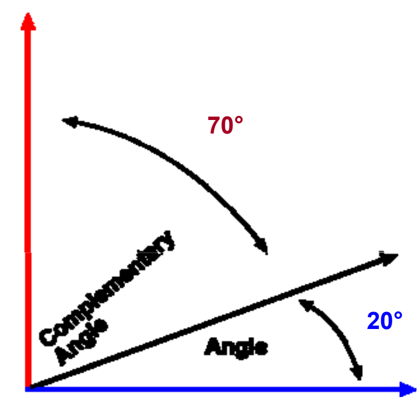
Definition: Complementary Angles

Complementary angles – n. two acute angles that add up to 90° or π/2 radians.



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Example: Complementary Angles

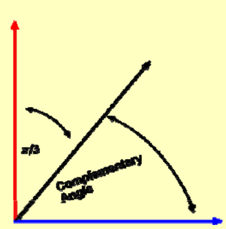
What angle is the complement of π/3?

Solution:

$\pi/2 - \pi/3 = 3\pi/6 - 2\pi/6$

$= (3 - 2)\pi/6$

$= \pi/6$



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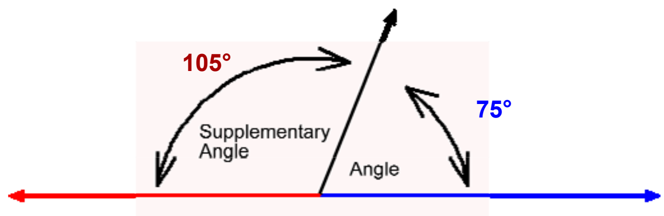
Definition: Supplementary Angles

Supplementary angles – n. two angles that add up to 180° or π radians.



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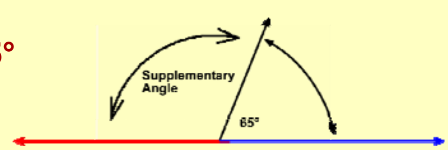
Example: Supplementary Angles

What angle is the supplement of 65°?

Solution:

180° – 65° = **115°**



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Measuring Angles

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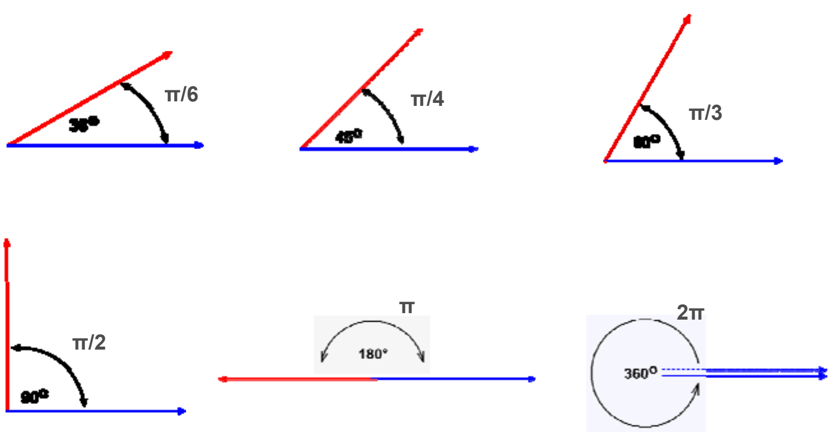
Common Angles

There are 6 angles commonly encountered in mathematics:



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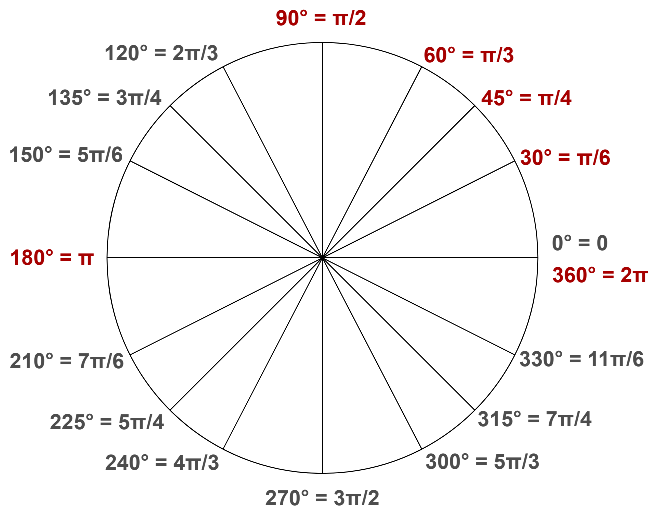
Common Angles and the Unit Cycle



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Example: Angles and Quadrants

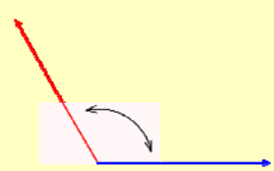
Which degree measurement describes an angle with a terminal side that lies in the second quadrant?

a) 50° **b) 130°** **c)210°**

Solution:

Quadrant II → (90°, 180°)

B. 130°



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Converting Degrees to Radians

Use the following formula to convert degrees into radians:

$$\text{degrees} * \frac{\pi}{180^\circ} = \text{radians}$$

Example: What is the radian equivalent of 20°?

$$20^\circ * \frac{\pi}{180^\circ} = \text{radians}$$

$$\frac{20^\circ \pi}{180^\circ} = \text{radians}$$

$$\frac{\pi}{9} = \text{radians}$$

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Converting Radians to Degrees

Use the following formula to covert radians to degrees:

$$\text{radians} * \frac{180^\circ}{\pi} = \text{degrees}$$

Example: What is the degree equivalent of 3π/4 radians?

$$\frac{3\pi}{4} * \frac{180^\circ}{\pi} = \text{degrees}$$

$$\frac{3 * 180^\circ}{4} = \text{degrees}$$

$$135^\circ = \text{degrees}$$

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Minutes and Seconds

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Converting Degrees to Minutes

Each degree converts into 60 minutes. Use the following formula to convert degrees to minutes:

Minutes = 60 · Degrees

Note: Minutes are denoted by a single quote. (ex. 20 minutes = 20')

Example: How many minutes are in 5°?

Minutes = 60 · 5°

= 300'

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Converting Minutes to Degrees

Use the following formula to convert minutes to degrees:

Degrees = Minutes/60

Example: How many degrees are in 120'?

Degrees = 120'/60

Degrees = 2°

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Converting Minutes to Seconds

Each minute converts into 60 seconds. Use the following formula to convert minutes to seconds:

Seconds = 60 · Minutes

Note: Seconds are denoted by a double quote. (ex. 15 seconds = 15")

Example: How many seconds are in 8'?

Seconds = 60 · 8'

= 480"

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Converting Seconds to Minutes

Use the following formula to convert seconds to minutes:

Minutes = Seconds/60

Example: How many minutes are in 240''?

Minutes = 240''/60

Minutes = 4'

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Decimals in Degrees

A degree measurement involving decimals can be rewritten as a combination of a degree measurement involving an integer and a minute measurement.

Example: Convert 80.025° into a combination of degrees and minutes.

$0.025° \cdot 60 = 1.5'$

80.025° = 80° 1.5'

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Decimals in Minutes

A minute measurement involving decimals can be rewritten as a combination of a minute measurement involving an integer and a second measurement.

Example: Convert 1.5' into a combination of minutes and seconds.

$$0.5' \cdot 60 = 30''$$

$$1.5' = 1'\,30''$$

$$\mathbf{80.025° = 80°\,1'\,30''}$$

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Arc Length and Sector

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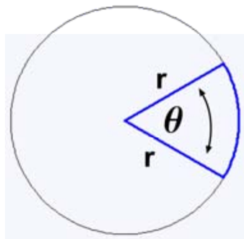
Arc Length

The length of an arc can be found using the formula:

$$s = r\theta$$

Where:

- s is the length of the arc
- r is the radius of the circle
- θ is the angle between the two radii



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Example: Arc Length

Find the length of the arc created by a circle of radius 2 and a central angle measuring π/6.

Solution:

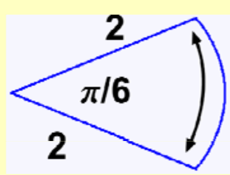
$s = r\theta$

$r = 2 \qquad \theta = \pi/6$

$s = 2 \cdot \pi/6$

$s = 2\pi/6$

$s = \pi/3$



$s = \pi/3$

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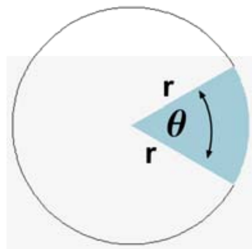
Area of a Sector

The **area of a sector** can be found using the formula:

$$A = \frac{1}{2} r^2\theta$$

Where:

- A is the area of the sector
- r is the radius of the circle
- θ is the angle between the two radii in radians



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Example: Area of a Sector

Find the area of the sector created from a circle of radius 5 and a central angle measuring 72°.

Solution:

$$72^\circ * \frac{\pi}{180^\circ} = \text{radians}$$

$$\frac{72^\circ \pi}{180^\circ} = \text{radians}$$

$$\frac{2\pi}{5} = \text{radians}$$

$A = \frac{1}{2}(5^2)(2\pi/5)$

$A = \frac{1}{2}(25)(2\pi/5)$

$A = \frac{1}{2}(50\pi/5)$

$A = 50\pi/10$

$A = 5\pi$

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Learning Summary

Positive angles are measured counter-clockwise; **negative angles** are measured clockwise.

The unit circle has a **radius of 1**.

The formulas for **arc length** and **area of a sector** require angles be in radian form.

Angles in standard position have an **initial side on the positive *x*-axis**.

Decimal parts of degree measures can be rewritten as **minutes and seconds**.

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Congratulations

You have successfully completed the core tutorial

The Unit Circle and Angle Measure

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What's Next …
Step 1: Concepts – Core Tutorial (Just Completed)
→ Step 2: Practice – Interactive Problem Drill
Step 3: Recap – Super Review Cheat Sheet
Go for it!
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