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Teach Yourself College Algebra in 24 Hours

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Logarithms

College Algebra Rapid Learning Series

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Learning Objectives

By completing this tutorial, you will be able to:

- Convert logarithmic equations to exponential equations.
- Use properties to evaluate, expand, and condense logarithmic expressions.
- Use the properties of logarithms to solve logarithmic equations.
- Solve exponential equations using logarithms.

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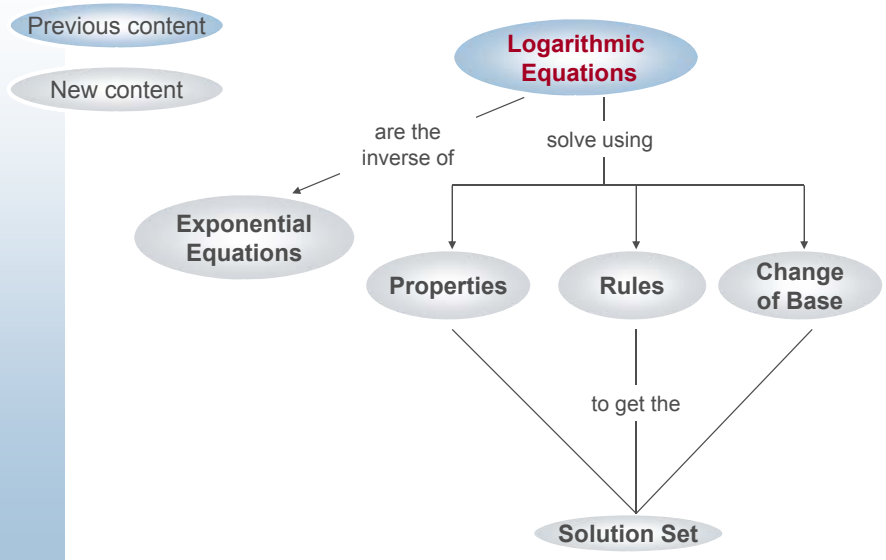
Concept Map



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Logarithm Basics

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What's a Logarithm?

Logarithm – The number of times a base must be multiplied by itself to reach a given number.

# of times "b" is multiplied

$$x = \log_b y$$

base

trying to reach

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Logarithm Algebra

If $x = \log_b y$

Then $y = b^x$

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Base 10 Logarithms

If no base is specified…

$$x = \log y$$

Then the base is “10”

$$x = \log_{10} y$$

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What's a Natural Logarithm?

Natural Logarithm (ln)
– Logarithm with "*e*" as the base; *e* ≈ 2.718.

Or

$$x = \log_e y$$

$$x = \ln y$$

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Logarithmic Equations Overview

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Definition – Logarithmic Equation

Logarithmic equation – The inverse of an exponential equation with base *b*.

For $x > 0$ and $b > 0$, $b \neq 1$,

$$\log_b x = y \text{ is the inverse of } b^y = x$$

Example: Write $\log_2 8 = 3$ as an exponential equation.

$$\log_2 8 = 3 \text{ is the inverse of } 2^3 = 8$$

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Examples – Logarithmic Equations

Let's look at some examples of logarithmic equations:

$\log_7 49 = 2$ is the inverse of $7^2 = 49$

$\log_x 125 = 3$ is the inverse of $x^3 = 125$

$\log_7 5 = 2x + 3$ is the inverse of $7^{2x+3} = 5$

$\log_{4x} 12 = 3$ is the inverse of $(4x)^3 = 12$

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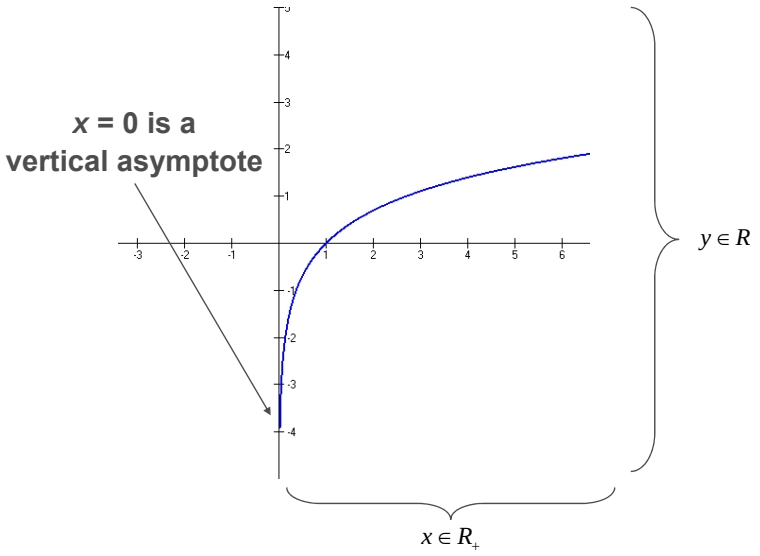
Graph: Logarithmic Function $b > 1$



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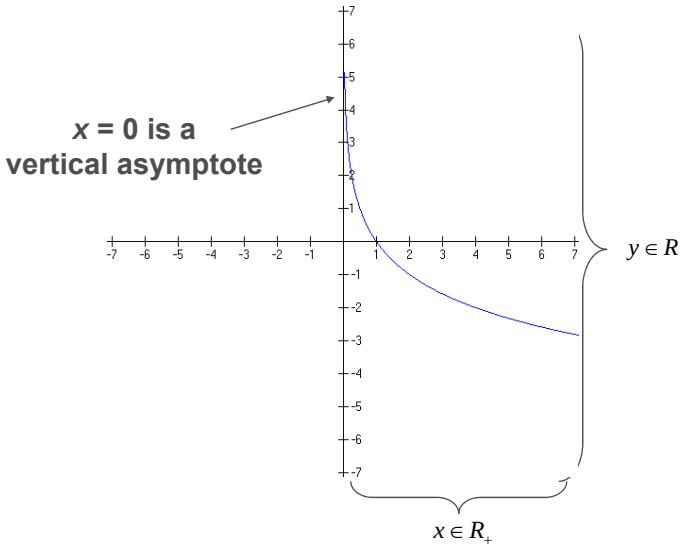
Graph: Logarithmic Function $b < 1$



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Basic Logarithmic Properties

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Identity Property

Identity Property

$$\log_b b = 1$$

Examples:

1) Evaluate $\log_8 8$.

$\log_8 8 = 1$ because $8^1 = 8$

2) Evaluate $\log_{7x} 7x$.

$\log_{7x} 7x = 1$ because $(7x)^1 = 7x$

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Zero Property

Zero Property

$$\log_b 1 = 0$$

Examples:

1) Evaluate $\log_9 1$.

$\log_9 1 = 0$ because $9^0 = 1$

2) Evaluate $\log_{7x} 1$.

$\log_{7x} 1 = 0$ because $(7x)^0 = 1$

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Inverse Property #1

Inverse Property #1

For $b > 0$ and $b \neq 1$,

$$\log_b b^x = x$$

Examples:

1) Evaluate $\log_5 5^6$.

$\log_5 5^6 = 6$ because $5^6 = 5^6$

2) Evaluate $\log_x x^3$.

$\log_x x^3 = 3$ because $x^3 = x^3$

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Inverse Property #2

Inverse Property #2

For $b > 0$ and $b \neq 1$,

$$b^{\log_b x} = x$$

Example: Evaluate $5^{\log_5 9}$.

$$5^{\log_5 9} = 9$$

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Common & Natural Logs

Logarithms with base 10 and *e*.

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Logarithms with Base 10

$\log_{10} x = \log x$

Examples:

1) Evaluate log 25.

Use the LOG key on your calculator.

$\log 25 \approx 1.398$

2) Evaluate log 100.

$\log 100 = 2$ because $10^2 = 100$

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Logarithms with Base *e*

$\log_e x = \ln x$

Examples:

1) Evaluate ln 25.

Use the LN key on your calculator.

$\ln 25 \approx 3.2189$

2) Evaluate ln 100.

$\ln 100 \approx 4.6052$

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Summary of Logarithmic Properties

This table lists the logarithmic properties.

Property	General	Common	Natural
Identity	$\log_b 1 = 0$	$\log 1 = 0$	$\ln 1 = 0$
Zero	$\log_b b = 1$	$\log 10 = 1$	$\ln e = 1$
Inverse #1	$\log_b b^x = x$	$\log 10^x = x$	$\ln e^x = x$
Inverse #2	$b^{\log_b x} = x$	$10^{\log x} = x$	$e^{\ln x} = x$

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Logarithmic Rules and Properties

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Product Rule

Product Rule

If *b*, *M*, and *N* are positive real numbers, and $b \neq 1$,

$$\log_b (MN) = \log_b M + \log_b N$$

Examples:

1) Expand $\log_5 (3 \cdot 25)$.

$$\log_5 (3 \cdot 25) = \log_5 3 + \log_5 25$$

2) Expand $\log_7 49x$.

$$\log_7 49x = \log_7 49 + \log_7 x$$

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Quotient Rule

Quotient Rule

If *b*, *M*, and *N* are positive real numbers, and $b \neq 1$,

$$\log_b (M / N) = \log_b M - \log_b N$$

Examples:

1) Expand $\log_5 (3 / 25)$.

$$\log_5 (3 / 25) = \log_5 3 - \log_5 25$$

2) Expand $\log_7 (49 / x)$.

$$\log_7 (49 / x) = \log_7 49 - \log_7 x$$

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Power Rule

Power Rule

If b and N are positive real numbers, and $b \neq 1$,

$$\log_b N^p = p \log_b N$$

Examples:

1) Expand $\log_5 25^3$.

$$\log_5 25^3 = 3 \log_5 25$$

2) Expand $\log_7 x^8$.

$$\log_7 x^8 = 8 \log_7 x$$

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Strategy to Remember the Rules

The strategy to remember these three rules is that

(1) **multiplication** becomes **addition**,

(2) **division** becomes **subtraction**, and

(3) a **power** becomes a **product**.

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Example: Expand Logarithm

Expand $\log_3 \frac{9x^4}{5^x}$.

Solution:

$$\log_3 \frac{9x^4}{5^x} = \log_3 9x^4 - \log_3 5^x$$

$$= \log_3 9 + \log_3 x^4 - \log_3 5^x$$

$$= \log_3 9 + 4\log_3 x - x\log_3 5$$

$$= \mathbf{2 + 4\log_3 x - x\log_3 5}$$

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Change of Base Property

$$\log_b N = \frac{\log N}{\log b} \quad \text{OR} \quad \log_b N = \frac{\ln N}{\ln b}$$

Example: Evaluate $\log_5 32$.

$$\log_5 32 = \frac{\log 32}{\log 5} \quad \text{OR} \quad \log_5 32 = \frac{\ln 32}{\ln 5}$$

$$\frac{\log 32}{\log 5} \approx 2.153$$

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Strategy to Remember Change of Base

In the change of base equation:

the **base** goes on the **bottom** of the fraction.

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Solve Equations Using Logarithms

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Example: Solve Using Product Rule

Solve the equation $\log_4 x + \log_4 (x + 6) = 2$.

Solution:

$\log_4 x + \log_4 (x + 6) = 2$

$\log_4 x(x + 6) = 2$

$\log_4 (x^2 + 6x) = 2$

$4^2 = x^2 + 6x$

$16 = x^2 + 6x$

$0 = x^2 + 6x - 16$

$0 = (x - 2)(x + 8)$

$x = 2, -8$

Check your answer!

By definition, you cannot take the log of a negative number, so -8 cannot be a solution.

$\boldsymbol{x = 2}$

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Example: Solve Using Quotient Rule

Solve the equation $\log_3 (x + 12) - \log_3 (x - 4) = 2$.

Solution:

$$\log_3 (x + 12) - \log_3 (x - 4) = 2$$

$$\log_3 \frac{(x + 12)}{(x - 4)} = 2$$

$$3^2 = \frac{(x + 12)}{(x - 4)}$$

$$9x - 36 = x + 12$$

$$8x = 48$$

$$\boldsymbol{x = 6}$$

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Solve Exponential Equations

To solve an exponential equation using logs:

(1) Isolate the expression containing the exponent.

(2) Take the log (or natural log) of both sides of the equation.

(3) Simplify using the power rule:

$\log_b N^p = p \log_b N$ or $\ln N^p = p \ln N$

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Example: Solve Exponential Equation

Solve the equation $2^x = 24$.

Solution:

$$2^x = 24$$

$$\log 2^x = \log 24$$

$$\frac{x \log 2}{\log 2} = \frac{\log 24}{\log 2}$$

$$x = \frac{\log 24}{\log 2}$$

$$x \approx 4.585$$

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