


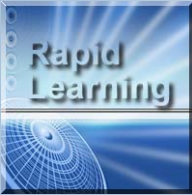
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 **Quadratic Equations**

**Intermediate Algebra Rapid Learning Series**

Wayne Huang, Ph.D.  
Mark Cowan, Ph.D.  
Diop El Moctar, Ph.D.  
Poornima Gowda, Ph.D.  
Daniel Deaconu, Ph.D.  
Fabio Mainardi, Ph.D.  
Theresa Johnson, M.Ed.  
Jessica Davis, M.S.  
Wendy Perry, M.A.  
Cesar Anchiraico, M.S.

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## Learning Objectives

By completing this core tutorial, you will:

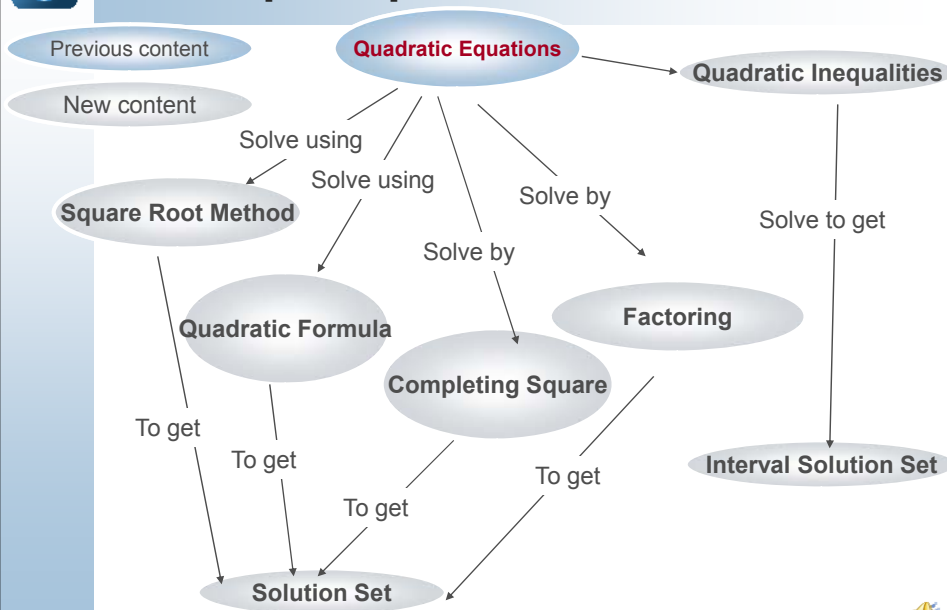


- Define quadratic functions.
- Identify the graphs of quadratic functions along with their properties.
- Solve quadratic equations by factoring, the square root method, completing the square and the quadratic formula.
- Use imaginary numbers in quadratic equation solutions.
- Identify and solve quadratic inequalities.

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## Concept Map

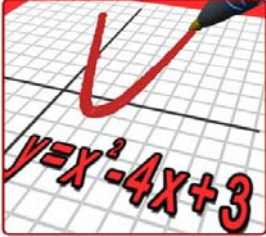


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## Quadratic Functions Defined



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## Definition – Quadratic Function

A **quadratic function** is a specific type of function written in the standard form:

$$f(x) = ax^2 + bx + c = 0, a \neq 0$$

Quadratic term

↖

linear term

↑

constant term

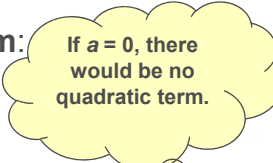

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Examples:

$$2x^2 - 2x + 1 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 1 = 0$$

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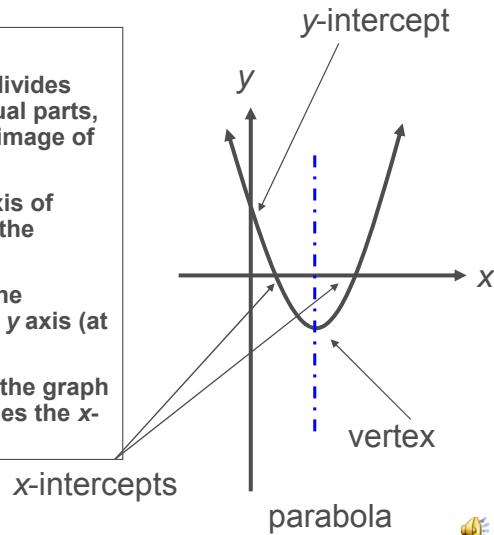


## Graph – Quadratic Function

The graph of a quadratic function is a **parabola**.

### ■ Graph Parts

- **Axis of Symmetry** - divides parabola into two equal parts, each part is a mirror image of the other.
- **Vertex** – where the axis of symmetry intersects the parabola.
- **Y-intercept** – where the parabola crosses the y axis (at  $x = 0$ ).
- **X-intercepts** – where the graph of the parabola crosses the x-axis (at  $y = 0$ )

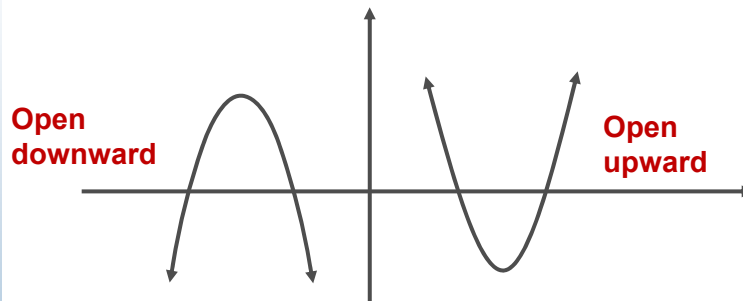


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## Parabola Properties – Open Up or Down

Parabolas may open **upward** (if  $a > 0$ ) or **downward** (if  $a < 0$ ).



Tip: Use your hands to help you remember the shape of a parabola.

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## The Mathematics of the Parabola

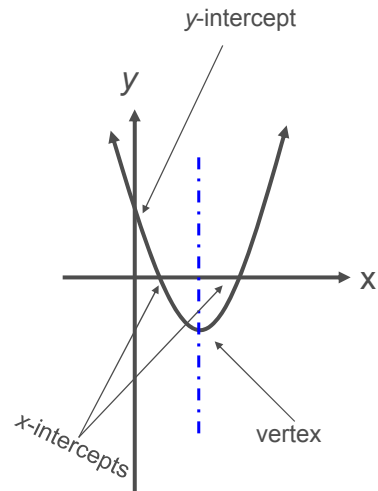
Given below are the formulas used to find the parts of the parabola.

■ **Axis of Symmetry:**  $x = -\frac{b}{2a}$

■ **Vertex:**  $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

■ **Y-intercept:**  $(0, c)$

■ **X-intercepts:**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



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## Maximum and Minimum

The vertex of a parabola can be a maximum or minimum value.

If  $a > 0$ , the vertex is a minimum.

If  $a < 0$ , the vertex is a maximum.



Examples:

$$3x^2 + 9x + 6 = 0$$


- $a > 0$ , this vertex is a minimum ( $3 > 0$ ).

$$-5x^2 + 6x + 1 = 0$$


- $a < 0$ , this vertex is a maximum ( $-5 < 0$ ).

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## Solving Quadratic Equations




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## Methods of Solving Quadratic Equations

**There are four methods of solving quadratic equations.**

- Factoring
- Square Root Method
- Completing the Square
- Using the Quadratic Formula



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## Factor an Expression

To factor an expression means to rewrite it as a product.

### Example

Factor:  $2x^2 - x$

$$2x^2 - x = x(2x - 1)$$



In this problem, we used the **Greatest Common Factor (GCF)**,  $x$ .

The GCF is the largest factor that elements in an expression have in common.

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## Zero Product Property

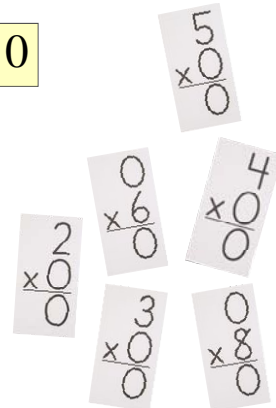
The **Zero Product Property** states that, if:

$$ab = 0, \text{ then } a = 0 \text{ or } b = 0$$

### Example

If  $(x + 4)(x - 2) = 0$ ,

then  $(x + 4) = 0$  or  $(x - 2) = 0$



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## Solve by Factoring

If the left side of an equation can be factored, then the zero-product property can be used to solve the quadratic equation.

**Example:** Solve  $x^2 + 2x = 15$  by factoring.

Rewrite equation in standard form:  $x^2 + 2x - 15 = 0$

Factor the left side:  $(x + 5)(x - 3) = 0$

Apply zero-product rule:  $x + 5 = 0$  or  $x - 3 = 0$

Solve for  $x$  in each equation:  $x = -5$  or  $x = 3$

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## Square Root Method

The square root method states that if  $A$  and  $B$  are algebraic expressions such that  $A^2 = B$ , then ...

$$A = \sqrt{B} \text{ or } A = -\sqrt{B}.$$

We can solve a quadratic equation by the square root method if we can write it in the form  $A^2 = B$ .



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## Solving by the Square Root Method

**Example:** Solve equation  $(3x-1)^2 - 9 = 0$ .

Apply square root method:  $(3x-1)^2 = 9$

$$3x-1 = \sqrt{9} \quad \text{or} \quad 3x-1 = -\sqrt{9}$$

$$3x-1 = 3 \quad \text{or} \quad 3x-1 = -3$$

Solve equations:  $3x-1+1 = 3+1$  or  $3x-1+1 = -3+1$

$$3x = 4 \quad \text{or} \quad 3x = -2$$

$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{2}{3}$$

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## Definition: Perfect Square Trinomial

**Perfect Square Trinomial** – A product obtained when a binomial is squared.

Example

$x^2 - 4x + 4$  can be written as  $(x-2)^2$

Properties of Perfect Square Trinomials:

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

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## The Difference of Two Squares

The difference of two squares can be written as:

$$a^2 - b^2 = (a - b)(a + b)$$



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## Completing the Square

**Completing the square involves finding the last term of a perfect square trinomial to solve an equation.**

Formula:  $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

### Example

Find the value to complete the square:  $x^2 + 6x + \underline{\quad}$ .

Calculate  $(b/2)^2$ :  $(6/2)^2 = 3^2 = 9$

This value completes the square:  $x^2 + 6x + \underline{9} = (x + 3)^2$

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## Steps to Completing the Square

### Completing the Square:

- 1 Find  $(b/2)^2$
- 2 Rewrite equation so that all  $x$  terms are on one side.
- 3 Complete the square by adding  $(b/2)^2$  to both sides.
- 4 Factor the perfect square trinomial.
- 5 Apply the square root method.
- 6 Solve for  $x$ .

$$x^2 - 10x + 13 = 0$$

- 1  $(-10/2)^2 = 25$
- 2  $x^2 - 10x = -13$
- 3  $x^2 - 10x + 25 = -13 + 25$
- 4  $(x - 5)^2 = 12$
- 5  $(x - 5) = \sqrt{12}$
- 6  $x - 5 = -\sqrt{12}$  or  $x - 5 = \sqrt{12}$

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## Recall the Quadratic Formula

The quadratic formula helps us to find solutions to quadratic equations of the form  $ax^2 + bx + c = 0$ .

### Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: Be sure to plug in the values of  $a$ ,  $b$ , and  $c$  exactly as they are. Alterations can lead to strange, incorrect answers.

Example: Find the solutions of the quadratic equation

$$x^2 - 8x - 9.$$

1. Identify the coefficients of the given quadratic equation.

$$\begin{array}{ccc} x^2 & - & 8x & - & 9 \\ \downarrow & & \downarrow & & \downarrow \\ a & & b & & c \end{array}$$

$$a = 1, b = -8, c = -9$$

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2. Plug the coefficients into the formula.

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-9)}}{2(1)}$$

3. Simplify the formula.

$$\frac{8 \pm \sqrt{100}}{2} = \frac{8 \pm 10}{2}$$

4. Find solutions.

$$\frac{8+10}{2} = \frac{18}{2} = 9 \quad \frac{8-10}{2} = \frac{-2}{2} = -1$$

The solutions are 9 and -1



## Solutions to Quadratic Equations

The number and type of solutions of a quadratic equation depends on the sign of the discriminant,  $b^2 - 4ac$ .

- If  $b^2 - 4ac > 0$ , then the equation has two distinct real solutions:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$x_1$  and  $x_2$  are the **roots** of the quadratic equation

- If  $b^2 - 4ac = 0$ , then the equation has one real solution:

$$x = -\frac{b}{2a}$$

- If  $b^2 - 4ac < 0$ , then the equation has two imaginary solutions (no real solutions) : What are imaginary solutions?

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## Complex or Imaginary Numbers

**Imaginary numbers** – Numbers of the form  $a + bi$ , where  $a$  and  $b$  are real numbers ( $b \neq 0$ ).

■  $bi$  indicates the part of the expression that is imaginary.

■  $i^2 = -1$

Example:  $\sqrt{-9} = i\sqrt{9} = i \cdot 3 = 3i$

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## Solving with the Quadratic Formula

**Example:** Solve equation  $2x^2 - x - 3 = 0$ .

- Identify  $a$ ,  $b$  and  $c$   $a = 2$ ,  $b = -1$  and  $c = -3$
- Evaluate  $b^2 - 4ac$   $(-1)^2 - 4 \times 2 \times (-3) = 25$
- Plug  $a$ ,  $b$  and  $c$  in quadratic formula

$$x = \frac{-(-1) + \sqrt{(-1)^2 - 4(2)(-3)}}{2 \times 2} \quad \text{or} \quad x = \frac{-(-1) - \sqrt{(-1)^2 - 4(2)(-3)}}{2 \times 2}$$

$$x = \frac{1 + \sqrt{25}}{4} = \frac{6}{4} \quad \text{or} \quad x = \frac{1 - \sqrt{25}}{4} = -\frac{4}{4}$$

- Solution  $x = \frac{3}{2}$  or  $x = -1$

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## Stop-and-Think

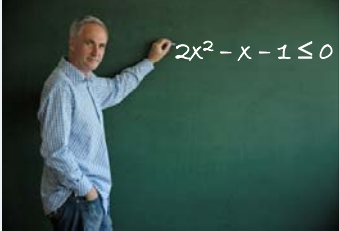
- 1) **True or false** : Quadratic equation  $ax^2 + bx + c = 0$  has two distinct roots if  $a^2 - 4bc > 0$ .  
**False. The discriminant is  $b^2 - 4ac$ .**
- 2) **True or false** : Quadratic equation  $ax^2 + bx + c = 0$  has two distinct roots if  $b^2 - 4ac > 0$ .  
**True.**
- 3) **True or false** : If  $b^2 - 4ac = 0$  then Quadratic equation  $ax^2 + bx + c = 0$  has no roots.  
**False. If  $b^2 - 4ac = 0$ , there is one root or solution.**
- 4) **True or false** :  $2x + 1 = 0$  is a quadratic equation.  
**False. This equation has no quadratic term.**


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## Quadratic Inequalities



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### Definition- Quadratic Inequality

**Quadratic Inequality** – A quadratic equation where the equal sign is replaced by an inequality symbol.

Examples:

$$2x^2 - x + 1 \geq 0$$
$$x^2 - 2x + 1 \leq 0$$

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## Solving a Quadratic Inequality

**Solving a quadratic inequality:**

- 1 Put the inequality in standard form.
- 2 Define boundaries.
- 3 Denote the test interval.
- 4 Find the sign (-, +) in every interval.
- 5 Determine which interval satisfies the quadratic inequality.



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## Quadratic Inequality Example - 1

**Solve the following Quadratic Inequality:**  $x^2 + 3x < 18$

- 1 Put the equation in standard form.

$$x^2 + 3x - 18 < 0$$

- 2 Define boundaries.

- To define boundaries, change the inequality to an equality,  $x^2 + 3x - 18 = 0$
- Find the solution of the equation,  $(x + 6)(x - 3) = 0$
- $x = -6$  or  $3$

- 3 Denote the test intervals.

The test intervals are  $(-\infty, -6)$ ,  $(-6, 3)$ ,  $(3, \infty)$ .



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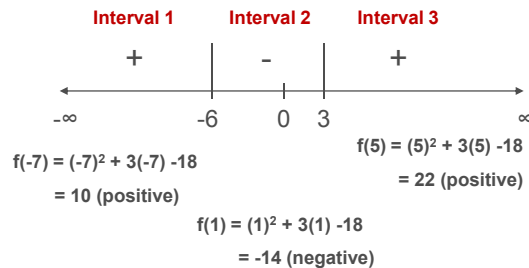


## Quadratic Inequality Example - 2

Solve the following Quadratic Inequality:  $x^2 + 3x - 18 < 0$

- 4 Find the sign (-, +) in every interval.

To find the sign in each interval, use a test value from the interval (plug this value in to the quadratic equation).



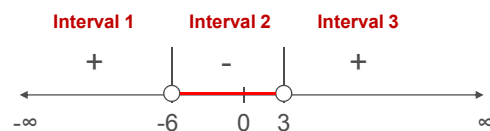
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## Quadratic Inequality Example - 3

Solve the following Quadratic Inequality:  $x^2 + 3x - 18 < 0$

- 5 Determine which interval satisfies the quadratic inequality.

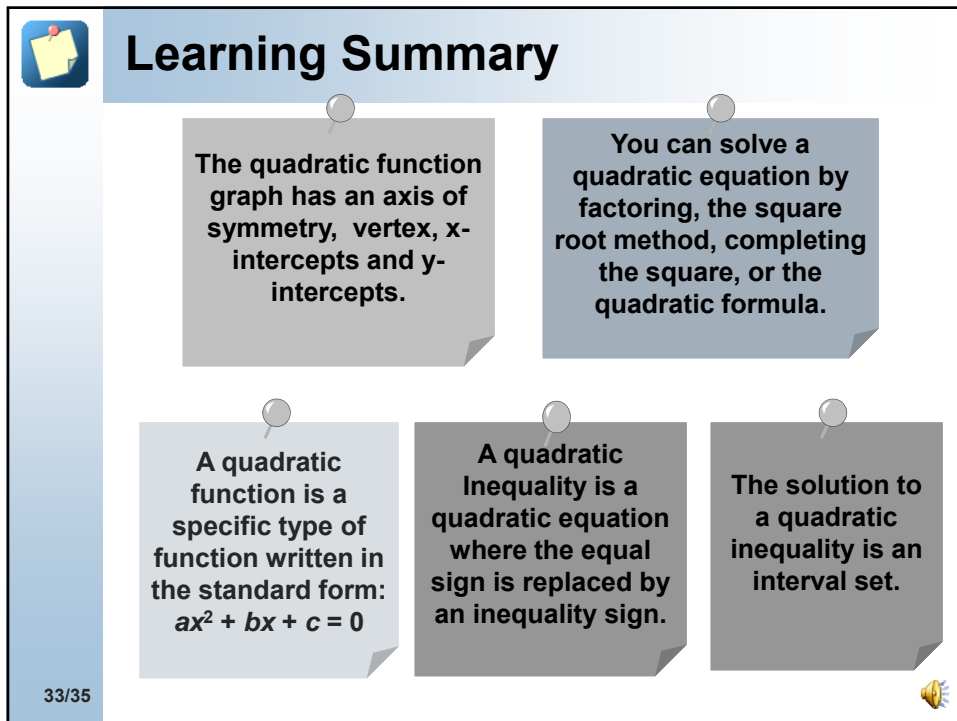


In our case, the quadratic inequality should be less than zero ( $< 0$ ).

The only interval satisfying this condition is interval 2,  $(-6, 3)$ .

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## Learning Summary


The quadratic function graph has an axis of symmetry, vertex, x-intercepts and y-intercepts.

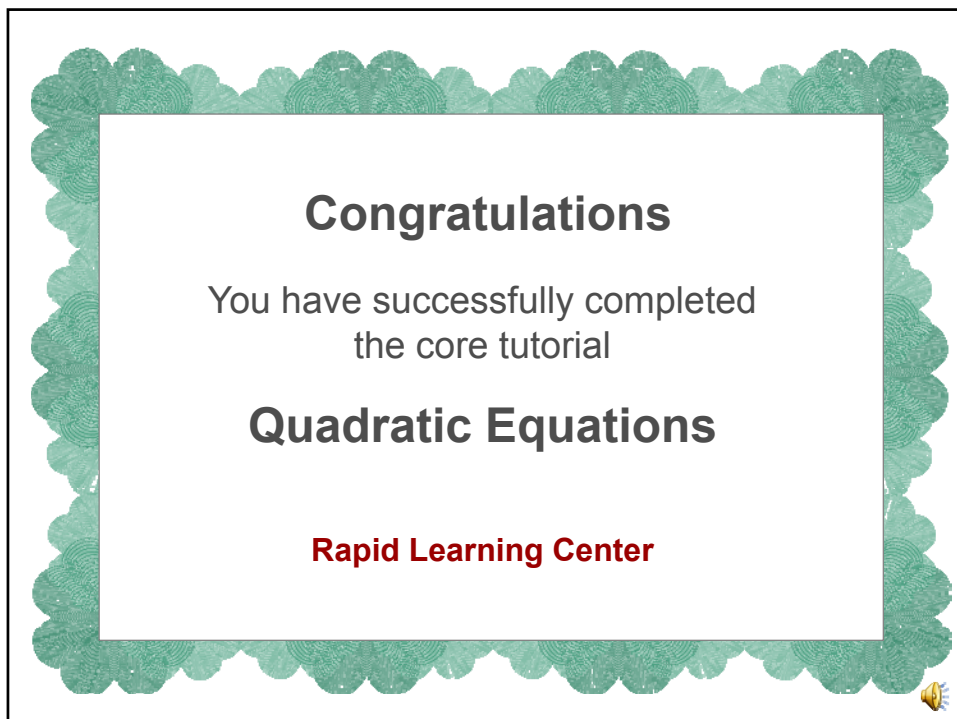
You can solve a quadratic equation by factoring, the square root method, completing the square, or the quadratic formula.

A quadratic function is a specific type of function written in the standard form:  
 $ax^2 + bx + c = 0$

A quadratic Inequality is a quadratic equation where the equal sign is replaced by an inequality sign.

The solution to a quadratic inequality is an interval set.

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## Congratulations

You have successfully completed  
the core tutorial

## Quadratic Equations


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**Go for it!**



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