Calculus Core Concept Cheat Sheet

06: Limits

Key Terms

- **The Limit of $f(x)$ as x Approaches a**: The value to which the output (dependent variable) of $f(x)$ approaches as x (independent variable) approaches the number a.
- **Velocity**: The rate and direction of an object's change in position with respect to time.
- **Average Velocity**: The change in an object's position divided by the amount of time that has elapsed.
- **Instantaneous Velocity**: The velocity of an object at one exact instant, measured using a limit called the **derivative**.
- **Convergent Infinite Series**: An unending list of numbers that adds up to a finite number.
- **Polynomial**: A mathematical expression consisting of one or more summed terms, each term consisting of a constant multiplier and one or more variables raised to integral powers.
- **Vertical Asymptote**: A vertical line that the graph of a function gets unboundedly close to but never reaches. The function is undefined at the x-value where the vertical asymptote occurs.
- **Removable Discontinuity**: A single point that the graph of a function "skips." Usually the result of having a common factor in the numerator and denominator of the function, which can be crossed out.
- **∞**: "Infinity" is not a number, but the idea of something greater than any positive real number. Similarly, −∞ is something less than any negative real number.

Approximating Limits Numerically

1. Identify the independent variable and the value it is approaching.

$$\lim_{t \to 4}\left(1 - t^2\right)$$

   In the above limit, the independent variable is t and it is approaching 4.
2. Pick two numbers, one slightly smaller and one slightly larger than the value the independent variable is approaching (within 0.01 often works; so try 3.99 and 4.01) and plug those numbers into the function.
3. If the results are relatively close to one another, pick a number in between the two as your approximation for the value of the limit.
4. If the results seem far apart, repeat steps 2 and 3 with numbers closer to 4, like 3.999 and 4.001. If the results are still far apart, the limit might not exist.

Finding Limits Algebraically – What Method Should I Use?

Plug the number the independent variable is approaching into the function and simplify. The results you get will determine what method you should use to find the limit. If you get…

- …a finite number, then that number is the limit. **Plugging in** was all you had to do!
- … $\frac{a}{0}$, where a is a finite number, then there is a **vertical asymptote** at the value x is approaching.
- … $\frac{0}{0}$, then there is a **removable discontinuity** at the value x is approaching.

Limits at Vertical Asymptotes

If you determine there is a vertical asymptote at the value x is approaching, follow these steps to find the limit:

- Pick numbers a little less and a little more than the value x is approaching (such as within 0.01)
- Plug these numbers into the function. You should get large positive or negative values.
- If both values are large positive numbers, the limit is ∞.
- If both values are large negative numbers, the limit is −∞.
- If one value is positive and one value is negative, the limit does not exist.

Limits at Removable Discontinuities

If you determine *f(x)* has a removable discontinuity at the value x is approaching, follow these steps to find the limit:

- Factor the numerator and denominator of the function.
- Look for a common factor in the numerator and denominator, and cancel it to simplify the function.
- Plug the value x is approaching back into the simplified function. The limit equals whatever number you get!

Limits – Problem Solving Tips

- A graph of the function will tell you a lot about the function. If you have access to one, or can sketch one yourself, you can use it to approximate the limit before applying a numerical or algegraic method.
- Identify the independent variable (often x, but not always) and the value it is approaching (a). This is found below the abbreviation "Lim" in the form $x \to a$.
- Plug the value the independent variable is approaching into the function to determine the method you should use (plugging in, vertical asymptote or removable discontinuity)
- If you can factor and simplify, do so!
- You can often find the limit of polynomials and many other simple functions by evaluating *f(x)* at $x = a$.

Typical Limit Problem

Evaluate the limit.

$$\lim_{x \to 1}\frac{x^2 - 1}{x^2 - 5x + 4}$$

Independent variable:
x is the independent variable, and it is approaching 1.

Plug in to determine method:

$$\frac{(1)^2 - 1}{(1)^2 - 5(1) + 4} = \frac{1 - 1}{1 - 5 + 4} = \frac{0}{0}$$ → Removable Discontinuity

Factor and simplify:

$$\frac{x^2 - 1}{x^2 - 5x + 4} = \frac{(x+1)\cancel{(x-1)}}{(x-4)\cancel{(x-1)}} = \frac{x+1}{x-4}$$

Plug back in:

$$\frac{(1)+1}{(1)-4} = \frac{2}{-3} = -\frac{2}{3}$$ The limit of the function as $x \to 1$ is -2/3!

How to Use This Cheat Sheet: These are the keys related this topic. Try to read through it carefully twice then rewrite it out on a blank sheet of paper. Review it again before the exams.

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