

13: Discrete Probability Distributions

Key Terms	Binomial Distribution Example
<ul style="list-style-type: none"> ■ Bernoulli trial: is an experiment whose outcome is random and can be either of two possible outcomes, "success" and "failure." ■ Independent trials: the outcome from one trial has no effect on the outcome to be obtained from any other trial. ■ Binomial Experiment: 1) predetermined n number of independent Bernoulli trials. 2) each trial result in only two outcomes: success and failure. 3) P(success)= p, is constant. ■ Geometric distribution: probability of the number of times needed to do something until the first successful outcome. The number of Bernoulli trials which must be conducted before a trial results in a success. ■ Geometric sequence: a sequence of numbers in which the (n+1)th number is a multiple of the nth number. P(X = n+1) is a multiple of P(X = n). 	<p>A random sample of 15 men is selected and the number who voted for Bush is recorded. Is this an example of a binomial experiment?</p> <ul style="list-style-type: none"> ■ n independent and identical trials: Trials: all men n=15 ■ Two outcomes, Success and Failure: Success=voted for Bush Failure=did not vote for Bush <ul style="list-style-type: none"> ■ Probability of success and failure: P(S)=0.4 P(F)=0.6 ■ x is the number of successes: x= number of men who voted for Bush <p>Yes, this is a binomial experiment</p>
Symbols	Comparison of Geometric and Binomial
<ul style="list-style-type: none"> ■ X = Binomial or Geometric random variable ■ x = number of successful trials ■ p = probability of a success ■ 1-p = probability of failure for a single trial ■ μ = mean ■ σ = standard deviation ■ σ² = variance ■ C = number of possible ways to have X = x ■ k = number of trials needed for first success 	<ul style="list-style-type: none"> ■ The geometric distribution is the only discrete memoryless random distribution. ■ It is a discrete analog of the exponential distribution. ■ This means that the chance of getting a heads up on the 7th trial after failing the first 6 times is the same probability as getting a heads on any of the first 6 trials. ■ The random process does not "remember" the number of failures. ■ In the <i>binomial distribution</i> we have fixed number of trials and a variable number of successes ■ In the <i>geometric distribution</i> we wait for a single success, but the number of trials is variable. ■ <i>Negative Binomial distribution</i> is the sum of Geometric distribution. How many trials will be needed to have the first "x" number of successes. <ul style="list-style-type: none"> □ If Y₁, ..., Y_r are independent geometrically distributed variables with parameter p, then $Z = \sum_{m=1}^r Y_m$ □ Follows a negative binomial distribution with parameters r and p.
Distribution Properties	Calculating Binomial Probability
<ul style="list-style-type: none"> ■ Bernoulli distribution: <ol style="list-style-type: none"> 1. μ = px(1) + (1-p)x(0) = p 2. σ² = pq ■ Binomial distribution: <ol style="list-style-type: none"> 1. $P(X = x) = C_x^n p^x (1-p)^{n-x}$ $= \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x},$ <p style="text-align: center;">where x = 0, 1, 2, ..., n, 0 < p < 1</p> 2. μ = np 3. σ² = npq = np(1-p) ■ Geometric distribution: <ul style="list-style-type: none"> ■ Probability: If the probability of success on each trial is p, then the probability that k trials are needed to get one success is either P(X=k) = (1-p)^{k-1}p for k = 1, 2, 3, ... ■ Mean: E(X) = 1/p ■ Variance: Var(X) = (1-p)/p² 	<p>A student claims that he gets grades better or equal to A, 40% of the time. This quarter, he gets only one A out of 4 courses. How likely is it that he got one A, or worse, out of four courses given his claim?</p> $P(x=0) = \frac{4!}{0!(4!)} (0.4)^0 (0.6)^4 = 1(1)(0.006) = 0.1296$ $P(x=1) = \frac{4!}{1!(3!)} (0.4)^1 (0.6)^3 = 4(0.4)(0.216) = 0.3456$ $P(x=0) + P(x=1) = 0.5616$
Geometric Distribution Example	Calculating Geometric Probability
<p>An experiment consists of rolling a single die. The event of interest is rolling a 2; this event is called a success. Is this a geometric experiment?</p> <ul style="list-style-type: none"> □ Rolling a 2 will represent a success, and rolling any other number will represent a failure. □ The probability of rolling a 2 on each roll is the same, p = 1/6. □ The observations are independent. □ A trial consists of rolling the die once. We roll the die until the first 2 appears. <p>Since all of the requirements are satisfied, this experiment describes a geometric setting</p>	<p>A child is trying to pick a yellow marble from a jar of 10 marbles with replacement and only 3 red marble in the jar. What is the probability of the girl succeeding in the 6th trial?</p> <p>P=0.3 P(X=6) = 0.3(1-0.3)⁶⁻¹ = 0.050</p>

How to Use This Cheat Sheet: These are the keys related this topic. Try to read through it carefully twice then write it out on a blank sheet of paper. Review it again before the exams.