
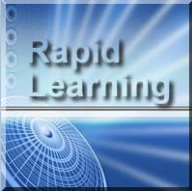
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 **Plane Geometry and Spatial Thinking**

Rapid Learning Tutorial Series

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Learning Objectives

After completing this tutorial, you will be able to:

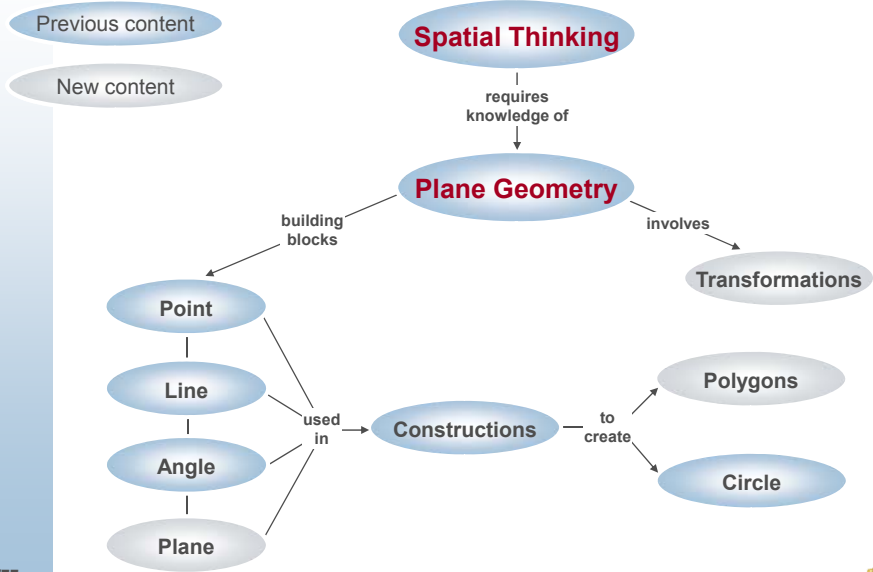


- Define point, line, plane and angle
- Define polygon and circle
- Construct geometric figures
- Perform transformations on polygons
- Use diagrams to solve problems

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Concept Map

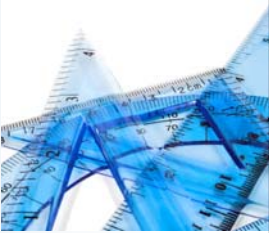


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Plane Geometry Basics



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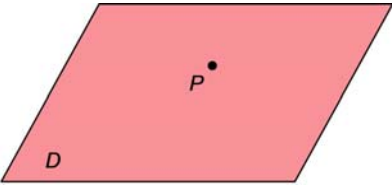


Definition: Point


Point – the result of the intersection of two lines; represented by a dot and labeled with a capital letter.

Point is the main building element of any figure in geometry. Any figure is the set of an infinite number of points.

A point has no dimensions; it has no length, no width, and no thickness.



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Definition: Plane Geometry

Plane geometry – the science of measurement; the geometry dealing with figures in a plane.



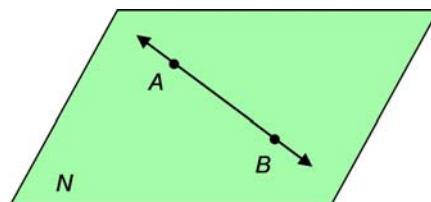
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Definition: Line

Line – a figure formed by connecting two points and extending beyond each point in both directions; represented with arrows at each end.

A line is formed when an infinite number of points lie next to each other in a straight path.



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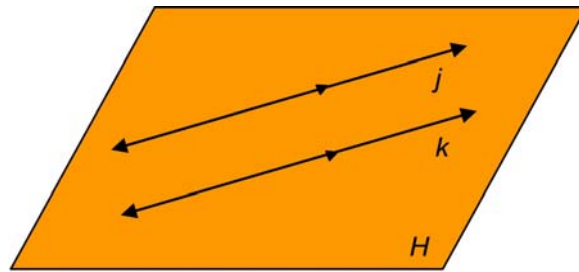




Definition: Parallel Lines

Parallel lines – two or more lines in the same plane that do not intersect.

Parallel lines share no common points.

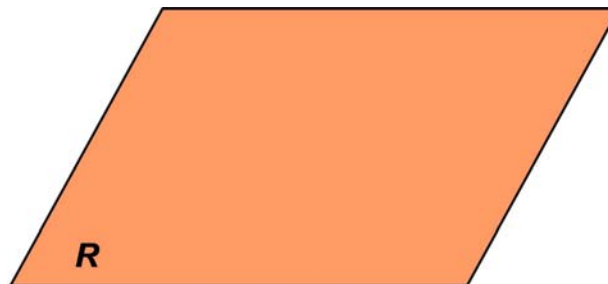


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Definition: Plane

Plane – a flat surface that extends indefinitely in all directions; represented by a parallelogram.



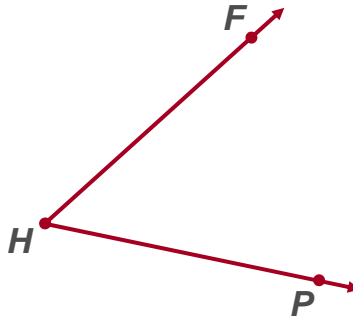
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Definition: Angle

Angle – a figure formed by two rays with a common initial point.



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Classification of Angles

Angles can be classified by their measures as one of the following:

- Acute
- Right
- Obtuse
- Straight



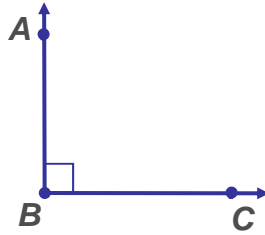
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Definition: Right Angle

Right angle – an angle with a measure of exactly 90° .

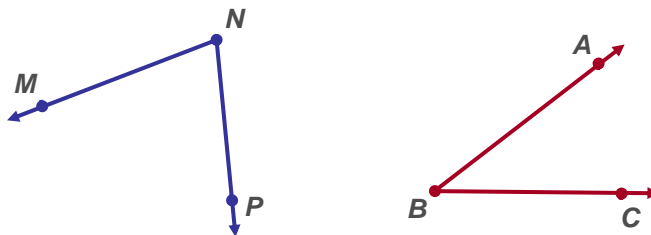


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Definition: Acute Angle

Acute angle – a positive angle with a measure less than 90° .



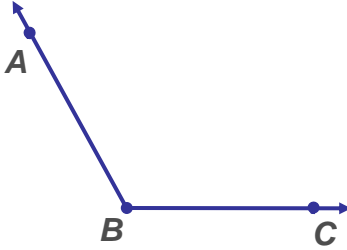
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Definition: Obtuse Angle

Obtuse angle – an angle with a measure between 90° and 180° .



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Definition: Straight Angle

Straight angle – an angle with a measure of exactly 180° .



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Example: Classifying Angles

What type of angle is represented by the corners of a regular sheet of paper?

- acute
- obtuse
- right
- straight



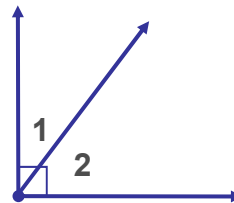
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Angle Pair Relationships

Two angles can have the following relationships:

- Vertical
- Complementary
- Supplementary
- Adjacent



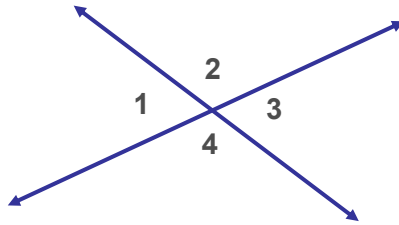
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Definition: Vertical Angles

Vertical angles – two angles that are across from each other at the intersection of two lines; they are always congruent.

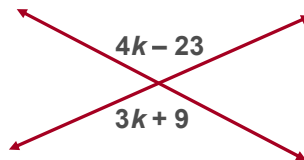


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Example: Vertical Angles

Using the vertical angles below, find the value of k .



Solution:

$$4k - 23 = 3k + 9$$

$$k - 23 = 9$$

$$k = 32$$

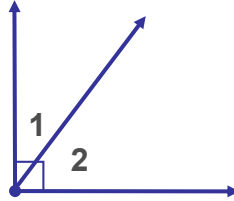
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Definition: Complementary Angles

Complementary angles – two angles whose sum is 90° .



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Example: Complementary Angles

$\angle 1$ and $\angle 3$ are complementary angles.
 $m\angle 3 = 62^\circ$. Find $m\angle 1$.

Solution:

$$m\angle 1 + m\angle 3 = 90^\circ$$

$$m\angle 1 + 62^\circ = 90^\circ$$

$$m\angle 1 = 28^\circ$$

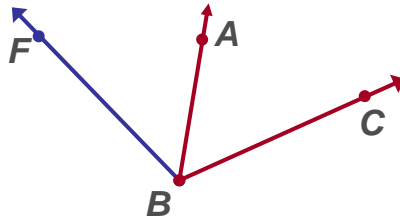
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Definition: Adjacent Angles

Adjacent angles – two angles that share a common side and a common vertex, but do not overlap.

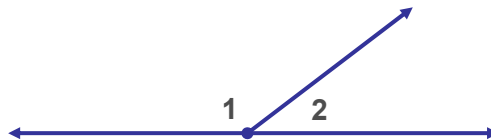


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Definition: Supplementary Angles

Supplementary angles – two angles whose sum is 180° .



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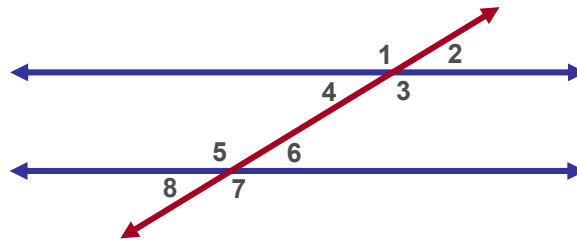




Parallel Lines and Transversals

Two coplanar lines are either intersecting or parallel.

When two parallel lines are intersected by a transversal, **eight angles** are formed around the points of intersection.

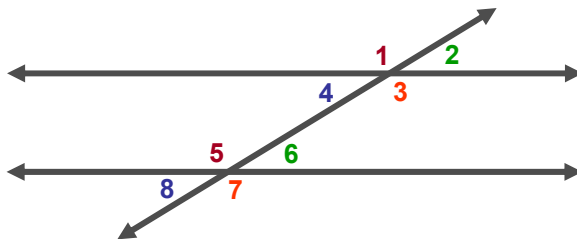


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Definition: Corresponding Angles

Corresponding angles – pairs of angles that are positioned the same at the intersection of two parallel lines and a transversal; they are congruent.



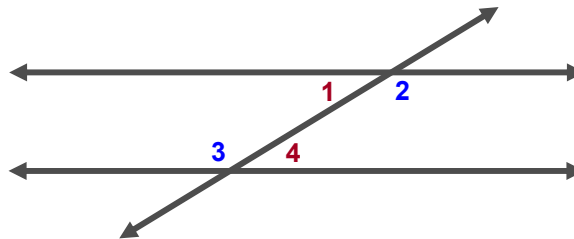
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Definition: Alternate Interior Angles

Alternate interior angles – pairs of angles located between the parallel lines on opposite sides of the transversal; they are congruent.

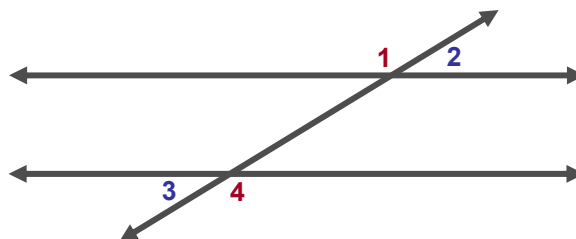


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Definition: Alternate Exterior Angles

Alternate exterior angles – pairs of angles located outside the parallel lines on opposite sides of the transversal; they are congruent.



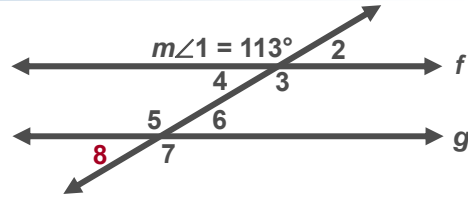
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Example: Parallel Lines

Lines f and g are parallel. Find $m\angle 8$.



Solution:

$$\angle 1 \cong \angle 5$$

$$m\angle 5 = 113^\circ$$

$$m\angle 5 + m\angle 8 = 180^\circ$$

$$113^\circ + m\angle 8 = 180^\circ$$

$$m\angle 8 = 67^\circ$$

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Polygons & Circles



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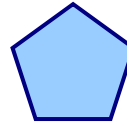
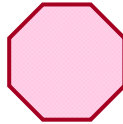
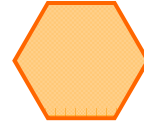
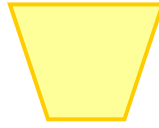
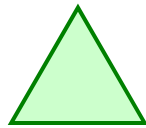




Definition: Polygon

Polygon – a closed plane figure with three or more sides; each side is a line segment.

Examples:



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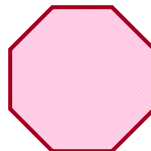
Definition: Regular Polygon

Regular polygon – a polygon where all sides are congruent and all angles are congruent.

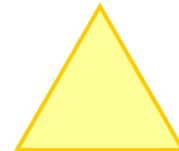
Examples:



Square



Regular Octagon



Equilateral Triangle

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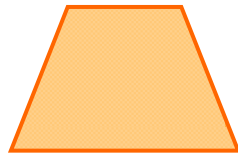




Definition: Irregular Polygon

Irregular polygon – a polygon where all the sides and angles are not congruent.

Examples:



Trapezoid



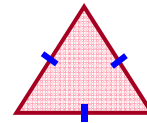
Scalene Triangle

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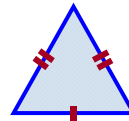


Classifying Triangles

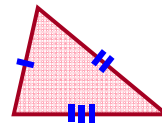
Equilateral triangle – a triangle with all sides congruent; also called equiangular.



Isosceles triangle – a triangle with at least two sides congruent.



Scalene triangle – a triangle with no two congruent sides; all interior angles have different measures.



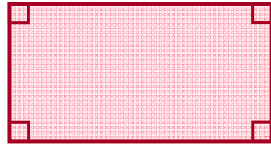
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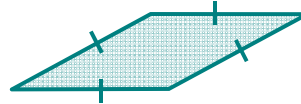


Definition: Parallelogram

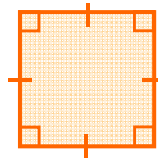
Parallelogram – a quadrilateral with two pairs of parallel sides.



Rectangle



Rhombus



Square

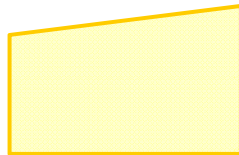
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Definition: Trapezoid

Trapezoid – a quadrilateral with one pair of parallel sides.

Isosceles trapezoid – a trapezoid with congruent legs.



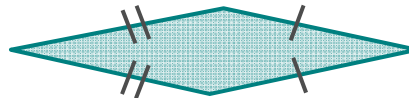
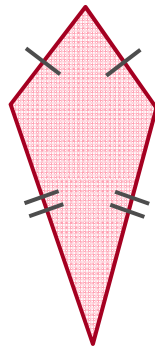
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Definition: Kite

Kite – a quadrilateral with no parallel sides and two pairs of adjacent sides that are congruent.

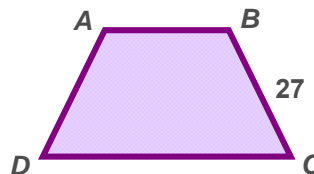


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Example: Isosceles Trapezoid Legs

Given $ABCD$ is an isosceles trapezoid and $BC = 27$, find the length of \overline{AD} .



Solution:

$$BC = 27$$

$$BC = AD$$

$$AD = 27$$

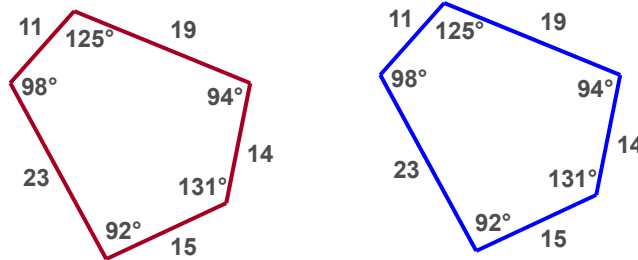
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Congruent Polygons

Two figures are congruent if their corresponding sides and angles are congruent.



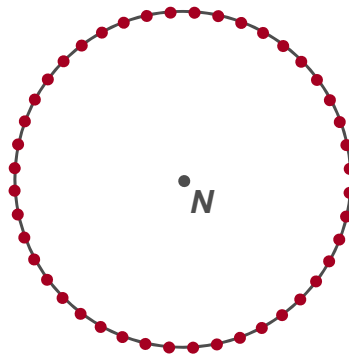
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Definition: Circle

Circle – a set of points that are a fixed distance from a given point (center).

Congruent circles have congruent radii.



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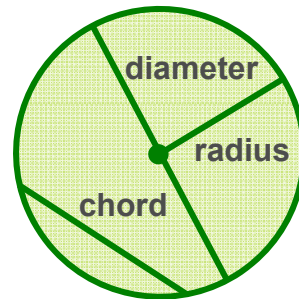


Circle Components

Chord – a line segment with both endpoints on the circle.

Radius – any segment that connects a point on a circle to the center of the circle; all radii of a circle have the same length.

Diameter – any segment that connects two points on a circle and passes through the center.

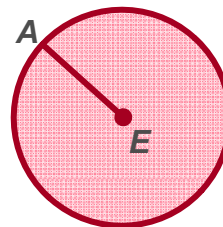
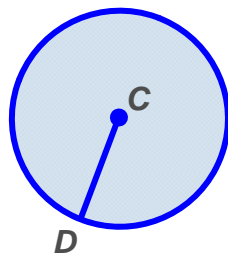


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Definition: Congruent Circles

Congruent circles – two circles that have radii or diameters of the same length.



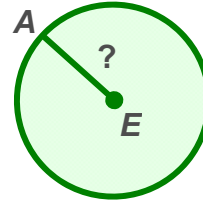
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Example: Congruent Circles

$\odot C$ is congruent to $\odot E$. The diameter d of $\odot C$ is 8 inches. What is the radius r of $\odot E$?



Solution:

$$\odot C \cong \odot E$$

$$\text{diameter of } \odot E = 8 \text{ in.}$$

$$\begin{aligned} \text{radius of } \odot E &= \frac{1}{2} \cdot \text{diameter} \\ &= \frac{1}{2} \cdot 8 \\ &= \mathbf{4 \text{ inches}} \end{aligned}$$

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Constructions



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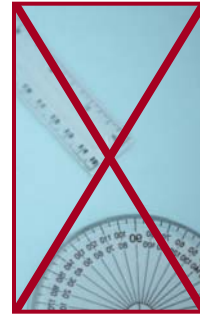


Tools of Construction

Rulers and protractors are not used in geometric constructions.

The only tools used when constructing geometric figures are:

- Straightedge
- Compass

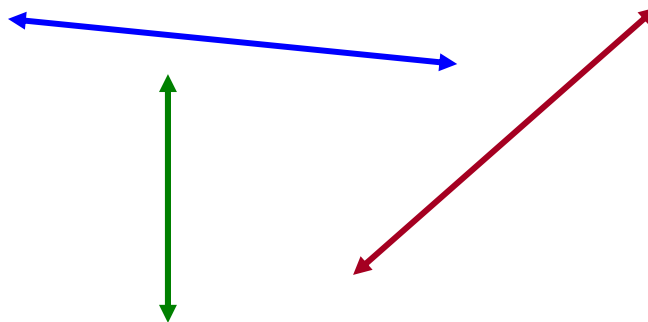


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Definition: Straightedge

Straightedge – any object that can be used to draw a straight line; not used to measure length.



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Definition: Compass

Compass – a tool used to draw circles and arcs of circles.

A compass has two arms joined at one end and free at the other. One arm is a **pointer** and the other arm has a pencil attached to it.



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Duplicating Segments – 1

Duplicate \overline{AB} .



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Duplicating Segments – 2

To construct a segment equal to \overline{AB} , draw an arbitrary segment, \overline{MN} , using a straightedge.

Make the length of \overline{MN} longer than the length of \overline{AB} .

Open a compass and place its pointer at A and its pencil at B .



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Duplicating Segments – 3

Keeping the compass with the same opening, place its pointer at M and draw an arc on \overline{MN} .

This arc intersects \overline{MN} at K . \overline{MK} is congruent to \overline{AB} .



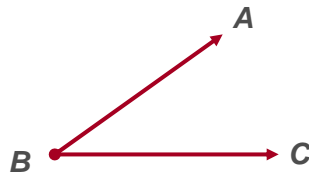
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Duplicating Angles – 1

Draw an angle congruent to $\angle ABC$.

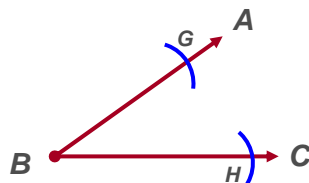


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Duplicating Angles – 2

Using a compass, draw two different arcs on \overrightarrow{BC} and \overrightarrow{BA} both centered at B . Denote the points of intersection by G and H , respectively.



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Duplicating Angles – 3

Draw arbitrary \overrightarrow{MN} . Using the compass, draw an arc centered at M whose radius is equal to \overline{BH} .



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Duplicating Angles – 4



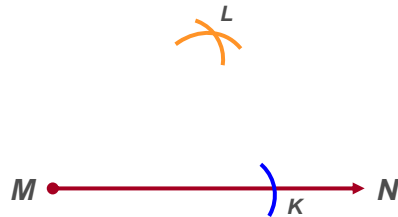
Set the compass radius equal to \overline{BG} . Place the pointer at M and draw an arc above \overrightarrow{MN} .

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Duplicating Angles – 5

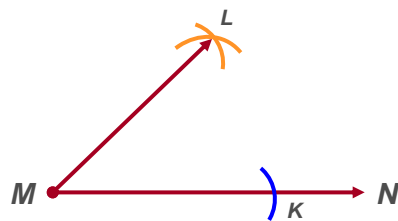


Open the compass as wide as the length of \overline{GH} . Place the pointer at K and draw an arc. This arc intersects the top arc at L .

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Duplicating Angles – 6



Draw a line through the points M and L . The result is $\angle LMN$, which is congruent to $\angle ABC$.

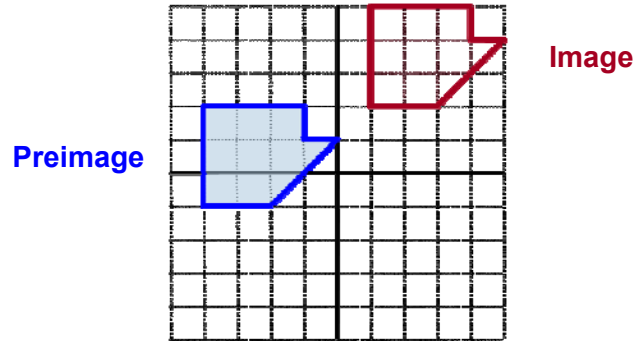
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Definition: Transformation

Transformation – a change in position, shape, or size of a figure.



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Transformations



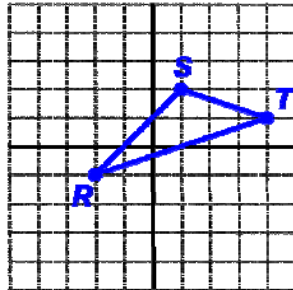
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Preimage Matrix

A matrix can be used to represent the preimage of a translation.



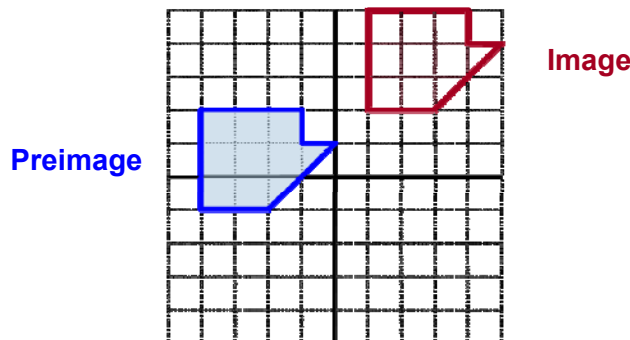
$$\begin{array}{l} \text{x-coordinate} \\ \text{y-coordinate} \end{array} \begin{array}{ccc} R & S & T \\ \begin{bmatrix} -2 & 1 & 4 \\ -1 & 2 & 1 \end{bmatrix} \end{array}$$

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Definition: Translation

Translation – a transformation that slides a figure to another location without any change in size or orientation.



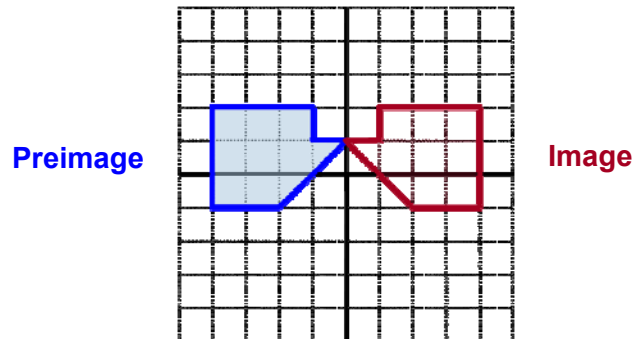
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Definition: Reflection

Reflection – a transformation that flips a figure over a line.



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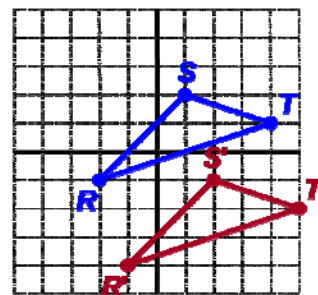


Translation Matrix

Matrix addition can be used to find the image of a translated figure.

Example:

- Add 1 to each x-coordinate.
- Add -3 to each y-coordinate.



$$\begin{array}{ccc}
 \text{Preimage} & \text{Translation} & \text{Image} \\
 \begin{bmatrix} -2 & 1 & 4 \\ -1 & 2 & 1 \end{bmatrix} & + \begin{bmatrix} 1 & 1 & 1 \\ -3 & -3 & -3 \end{bmatrix} & = \begin{bmatrix} -1 & 2 & 5 \\ -4 & -1 & -2 \end{bmatrix} \\
 R \quad S \quad T & & R' \quad S' \quad T'
 \end{array}$$

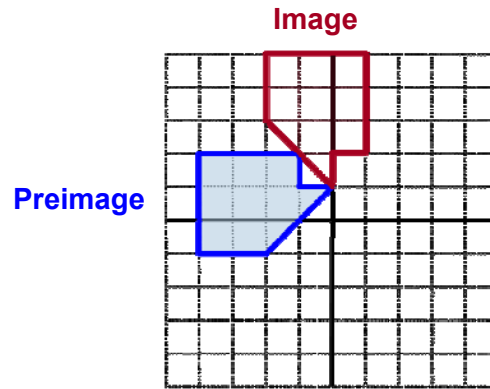
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Definition: Rotation

Rotation – a transformation that turns a figure around a fixed point.

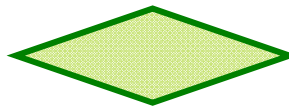
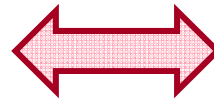
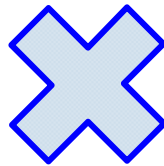


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Point Symmetry

If a figure can be mapped onto itself with a 180° rotation, then it has **point symmetry**.



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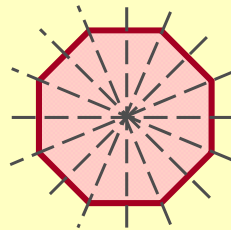
Example: Reflection Symmetry

How many lines of symmetry does a stop sign have?



Solution:

- 4 through the vertices
- 4 through the sides
- **8 lines of symmetry**

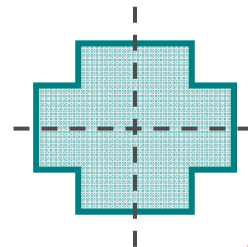
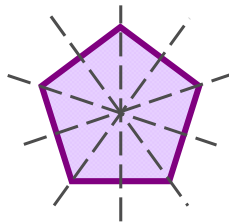
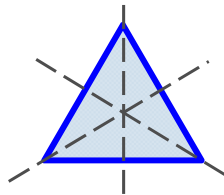
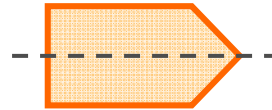
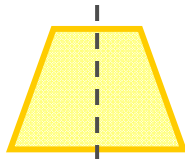


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Reflection Symmetry

If a figure can be folded in half so that the halves match exactly, then it has **reflection symmetry**.



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Definition: Tessellate

Tessellate – to cover a plane with identical shapes with no overlapping or gaps.

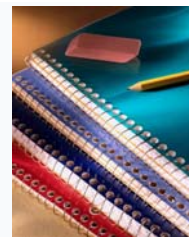
This tessellation of a trapezoid was created using a combination of transformations.



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Proof in Geometry



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Reasoning with Diagrams

Some problem situations are easier to understand when a related diagram is given.

Diagrams can be used to list known information and establish what information is needed to solve the problem.



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Reasoning Strategies

Inductive reasoning – a reasoning strategy that uses a set of examples to find a pattern to support a conclusion.

Deductive reasoning – a reasoning strategy that uses facts and theorems to arrive at a conclusion.



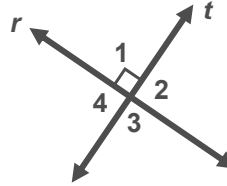
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Example 2: Proof Using a Diagram

Prove $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.



Solution:

$$\angle 1 \cong \angle 3 \rightarrow m\angle 3 = 90^\circ$$

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$90^\circ + m\angle 2 = 180^\circ$$

$$m\angle 2 = 90^\circ$$

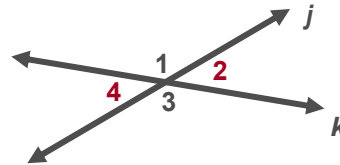
$$\angle 2 \cong \angle 4 \rightarrow m\angle 4 = 90^\circ$$

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Example 1: Proof Using a Diagram

Prove $\angle 2 \cong \angle 4$.



Solution:

$$m\angle 2 + m\angle 3 = 180^\circ \quad m\angle 3 + m\angle 4 = 180^\circ$$

$$m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$$

$$m\angle 2 = m\angle 4$$

$$\angle 2 \cong \angle 4$$

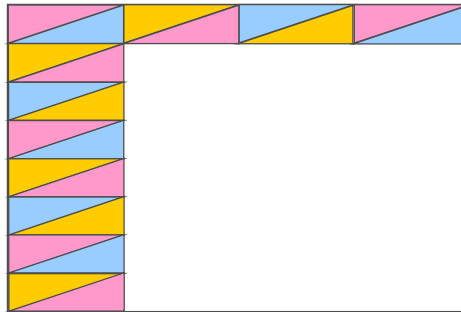
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Example 3: Proof Using a Diagram – 1

Justin is tiling an 8 in. × 12 in. surface with colored glass tiles. Each tile is a right triangle with a base of 1 in. and a height of 3 in. How many tiles will Justin need to tessellate (cover) the surface?



Row = 8 triangles

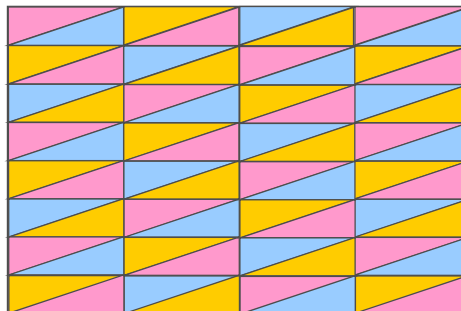
Column = 8 rows

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Example 3: Proof Using a Diagram – 2

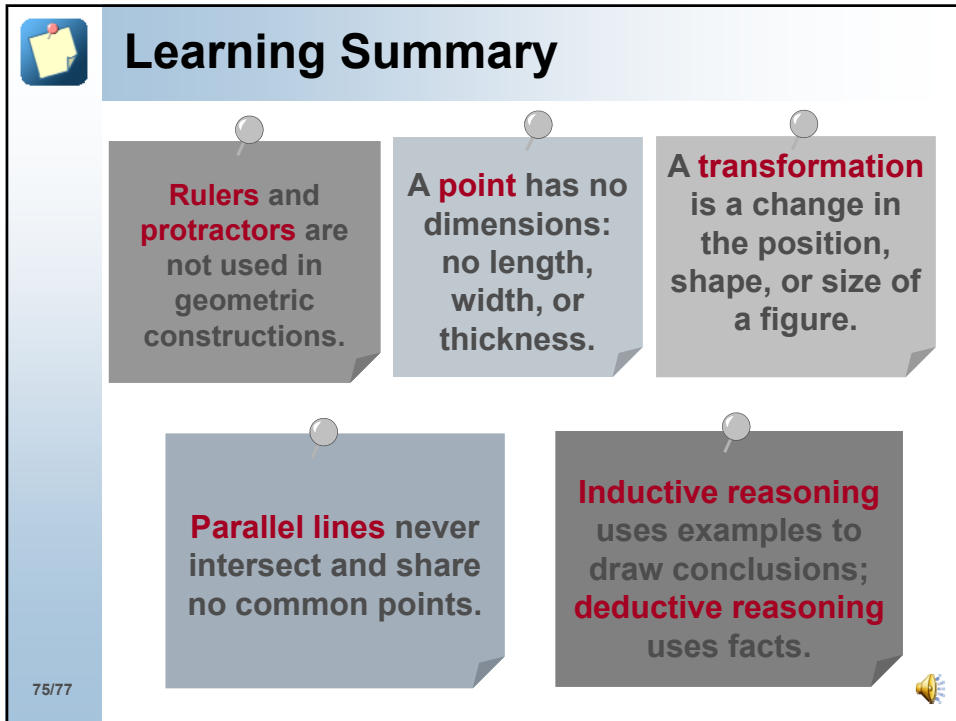
Justin is tiling an 8 in. × 12 in. surface with colored glass tiles. Each tile is a right triangle with a base of 1 in. and a height of 3 in. How many tiles will Justin need to tessellate (cover) the surface?



8 triangles × 8 rows = **64 triangles**

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Learning Summary


Rulers and **protractors** are not used in geometric constructions.

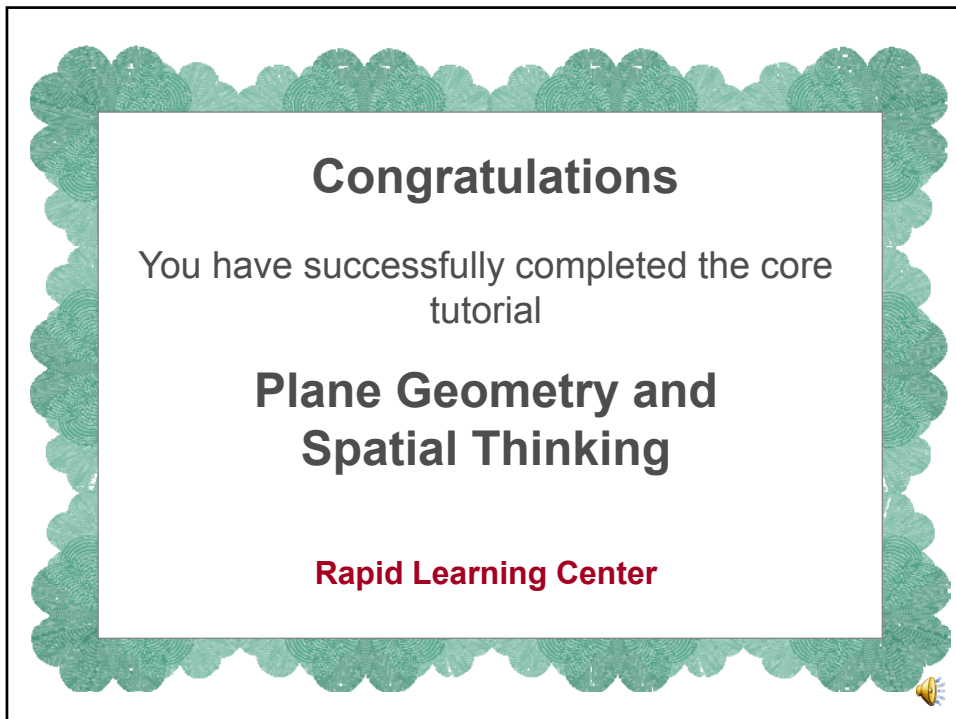
A **point** has no dimensions: no length, width, or thickness.

A **transformation** is a change in the position, shape, or size of a figure.

Parallel lines never intersect and share no common points.

Inductive reasoning uses examples to draw conclusions; **deductive reasoning** uses facts.

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Congratulations

You have successfully completed the core tutorial

Plane Geometry and Spatial Thinking


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Chemistry :: Biology :: Physics :: Math



What's Next ...

Step 1: Concepts – Core Tutorial (Just Completed)

→ Step 2: Practice – Interactive Problem Drill

Step 3: Recap – Super Review Cheat Sheet

Go for it!



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