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1/48

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Vectors and Kinematics in 2D

Physics Rapid Learning Series

Wayne Huang, Ph.D.
Keith Duda, M.Ed.
Peddi Prasad, Ph.D.
Gary Zhou, Ph.D.
Michelle Wedemeyer, Ph.D.
Sarah Hedges, Ph.D.

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2/48

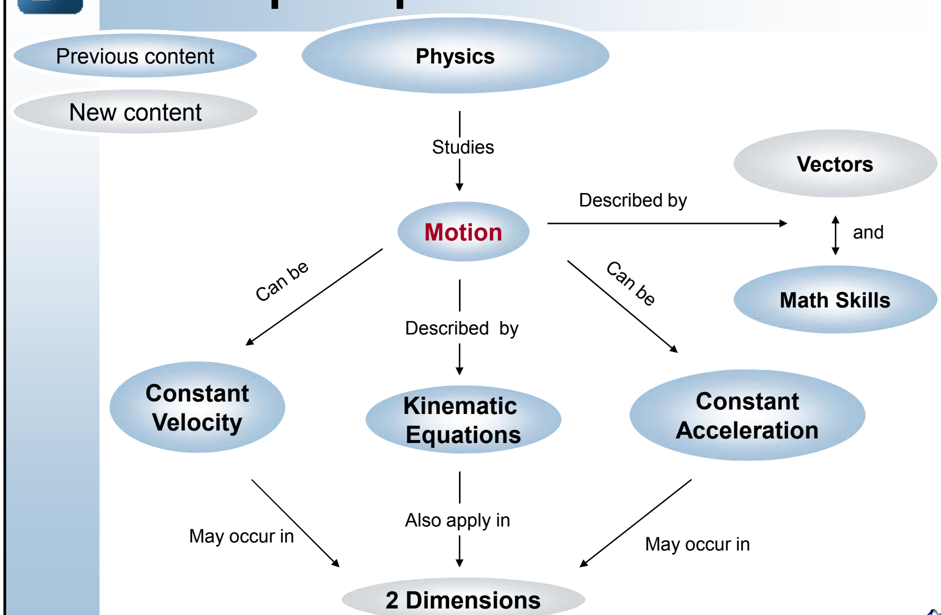
Learning Objectives

By completing this tutorial, you will:

- Understand vectors and scalars.
- Complete basic vector operations.
- Describe motion in 2 dimensions.
- Calculate various quantities involving projectile motion.

3/48

Concept Map



4/48

Vectors and Scalars

It is important to be able to distinguish between a vector and a scalar.

5/48

Vector Definition

Any measurement or quantity can be categorized into one of two types:

A vector is a quantity that has magnitude (size), and direction.

Ex: Velocity, displacement, acceleration

A scalar is a quantity that has only magnitude (size).

Ex: speed, distance, temperature, mass

6/48

Vector Examples

All of the following measurements involve magnitude and direction, thus they are vectors.

10 m/s East

$-9.8\ m/s^2$

The negative sign indicates the downward direction

50 m/s, 30 degrees above horizontal

7/48

Vector Notation

Due to their directional nature, vectors are usually drawn with arrows to signify their direction.

10km Southeast

Occasionally they are also drawn to scale, similar to the scale on maps.

1 cm = 100 m

8/48

Textbook Notation

In textbooks, vectors must be distinguished from scalars. This can be accomplished in a variety of ways.

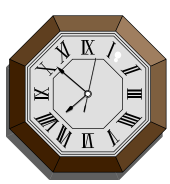
A vector is usually abbreviated by a letter that identifies it as a vector in one of the following ways:

A	$\vec{\mathbf{A}}$	$\bar{\mathbf{A}}$
Bold letter	Arrow above	Line above

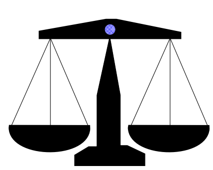
9/48

Scalar Examples

All of the following measurements involve only a size or magnitude. They are directionless scalars.



10 seconds



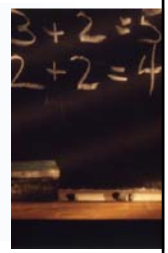
15 kg

35 miles per hour

(no direction)

10/48

Vector Addition



In order to add vectors, certain rules must be followed.

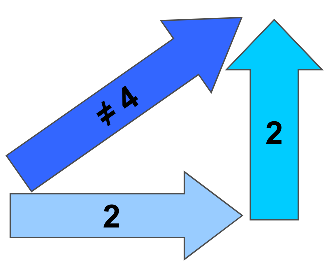
11/48

Scalar Addition vs Vector Addition

Just as scalar quantities can be added,

$$2 + 2 = 4$$

Vector quantities can be added also.
However, their rules are slightly different.



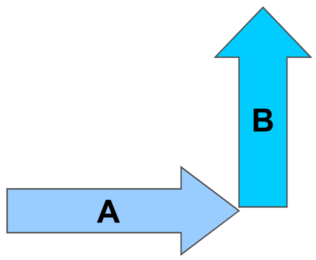
2 + 2 doesn't equal 4!

12/48

Vector Addition Rule

When vectors are graphically added, they are drawn **head** to **tail**.

This may also be described as placing the arrowhead of one vector next to the tail end of another vector:

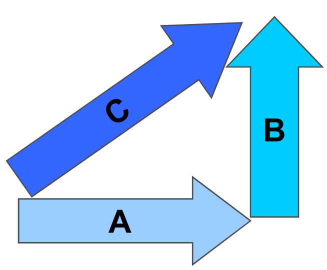


$$\vec{A} + \vec{B}$$

13/48

Resultant

When these vectors are added in this way, the sum, or **resultant**, is drawn from the tail of the first vector to the tip of the last vector.



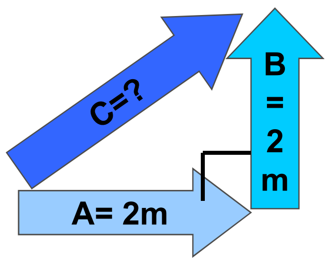
$$\vec{A} + \vec{B} = \vec{C}$$

14/48

Calculation of the Resultant

Mathematics can be used to calculate the magnitude and/or direction of the resultant. For the magnitude, use the Pythagorean theorem:

$$\vec{A} + \vec{B} = \vec{C}$$



$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{2^2 + 2^2}$$

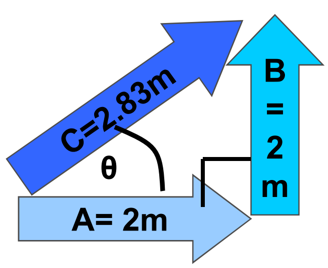
$$c = \sqrt{8} = 2.83\text{m}$$

15/48

Direction of the Resultant

Simple trig functions can be used to calculate the direction of the resultant.

$$\vec{A} + \vec{B} = \vec{C}$$



$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

B (opp), A (adj)

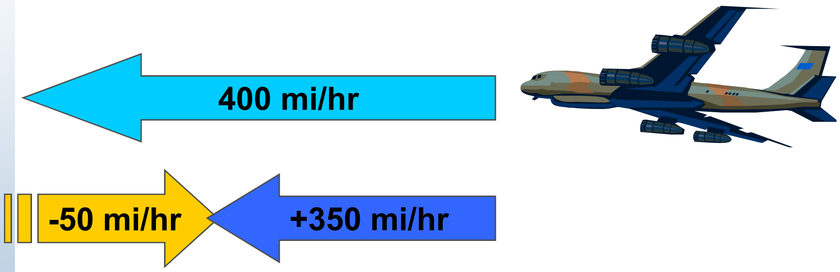
$$\tan\theta = \frac{2\text{m}}{2\text{m}} = 1$$

$$\theta = 45^\circ$$

16/48

Vectors Added in a Line

Sometimes, vectors are added along the same line. This simplifies things greatly.

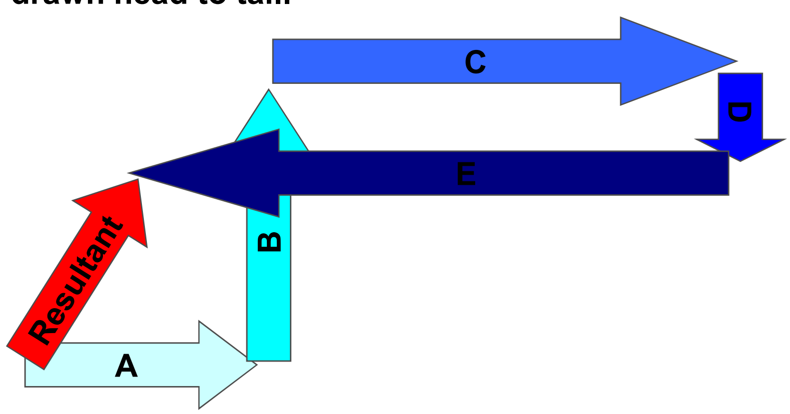


- The airplane flies due west at +400 mi/hr (or mph)
- The headwind blows due east at 50 mi/hr (negative)
- Adding the vectors tip to tail gives a resultant of +350 mi/hr.

17/48

Multiple Vectors Added

When multiple vectors are added, they are still drawn head to tail.



$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = \text{Resultant}$$

18/48

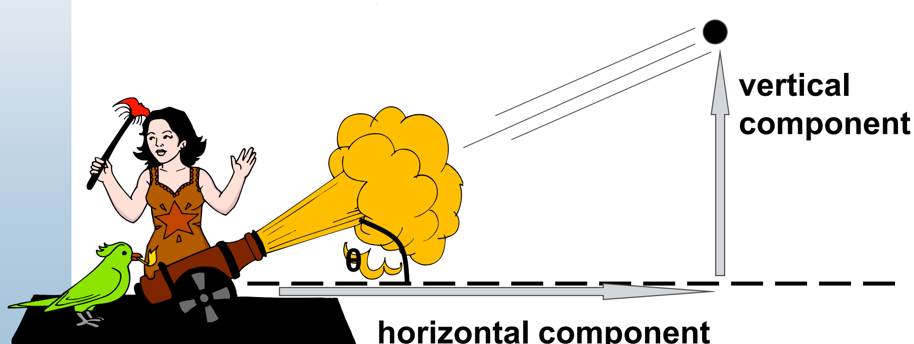
Motion in 2 Dimensions

Often, objects don't move at nice neat angles. There are ways to simplify these situations.

Diagonal Motions

A cannon is fired at some angle with the horizontal.

Its diagonal motion could be simplified by showing its motion only in the horizontal and vertical directions.

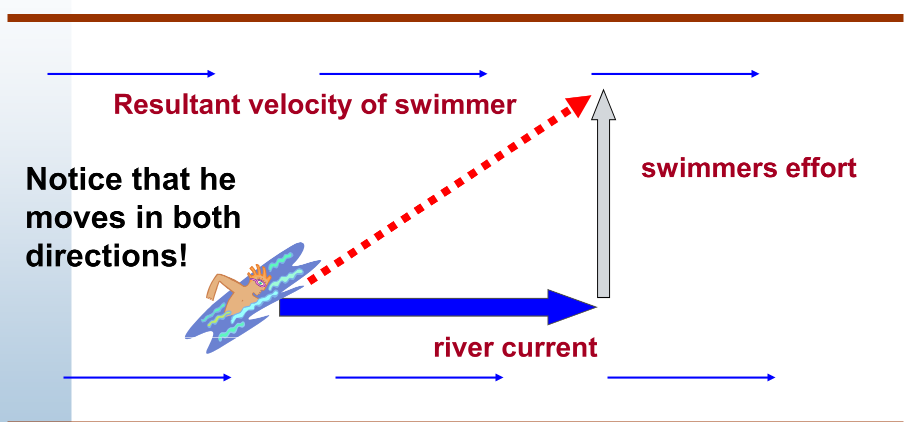


Defintion - Vector Component

Vector Component - **The parts into which a single vector can be separated and that act in different directions from the vector.**

21/48

Swimming Across a River



Swimming across the river involves motion in two separate perpendicular directions.

22/48

Projectile Motion

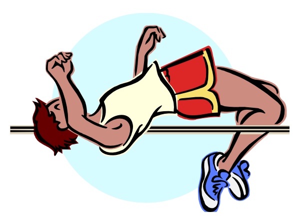
A projectile is any object moving through the air or space that is only acted on by gravity.

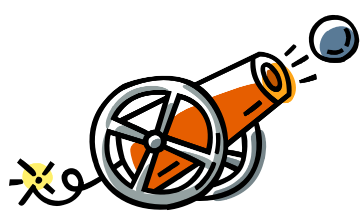
23/48

Projectile Examples

A projectile could take many forms:

- A cannon ball
- A high jumping athlete
- A ball thrown through the air

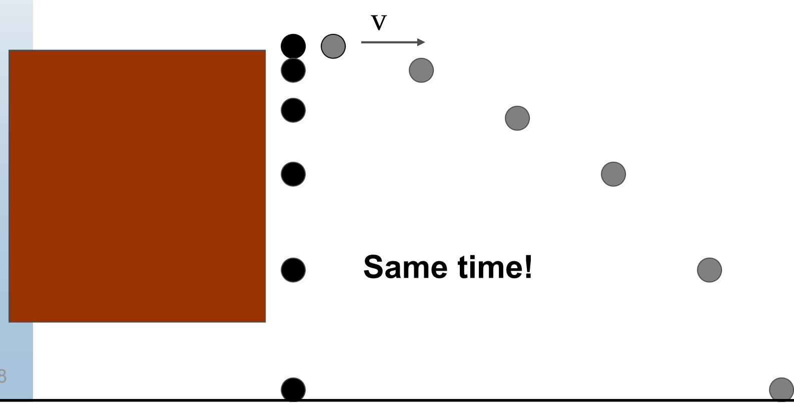




24/48

Classic Projectile Question

A ball is dropped vertically, and one is launched horizontally, each from the same height. Which one hits the ground first?



25/48

The Explanation

Why does it work this way?

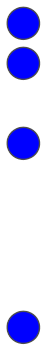
They both hit at the same time because they both fall the same vertical distance and experience the same acceleration due to gravity.

Although the thrown one has an additional horizontal velocity, that doesn't affect its vertical motion.

Breaking an objects motion into horizontal and vertical components is very useful!!

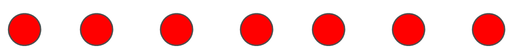
26/48

Further Explanation



In the vertical direction, both the balls experience the usual acceleration from gravity. Notice the increasing falling distance and speed in the vertical direction.

In the horizontal direction, the projected ball has a constant velocity. Thus, it covers regular, consistent horizontal distances.



27/48

Thought Question Explanation

Why does it work this way?

Both fall the same vertical distance, and are accelerated the same by gravity, thus the times would be equal.

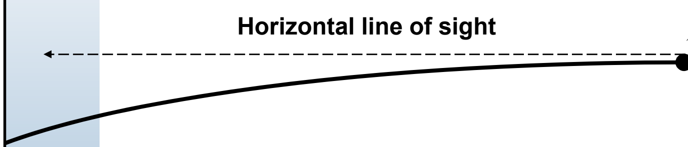
The fired bullet simply has a very large extra horizontal component.

This is usually never observed since it would require a large flat open area.

28/48

Thought Question:

At the same time that a high speed bullet is fired horizontally from a rifle, another bullet is simply dropped from the same height. Which bullet strikes the ground first?



If we could see far downrange, we would notice that they both hit at the same time!

29/48

Monkey Explanation

You might think the bullet would go above the monkey since he is falling.

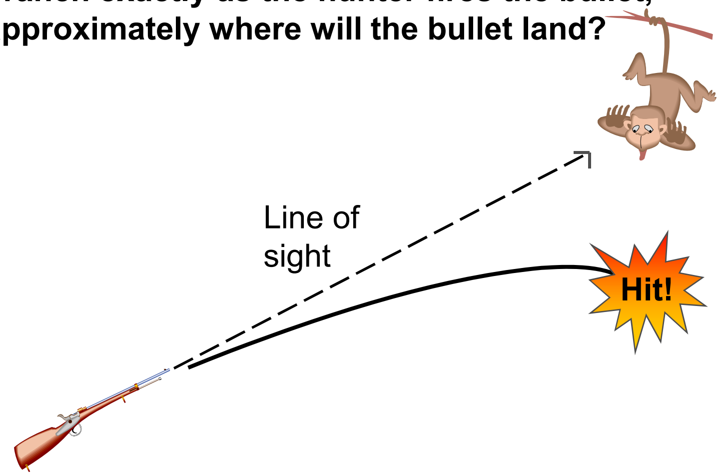
Since they are in the air for the same amount of time, the bullet hits the target!

However, the bullet is falling with the same vertical acceleration too.

30/48

The Monkey and the Hunter

A big game hunter spots a monkey in the tree and aims directly at him. If the monkey releases the branch exactly as the hunter fires the bullet, approximately where will the bullet land?



31/48

Formulas

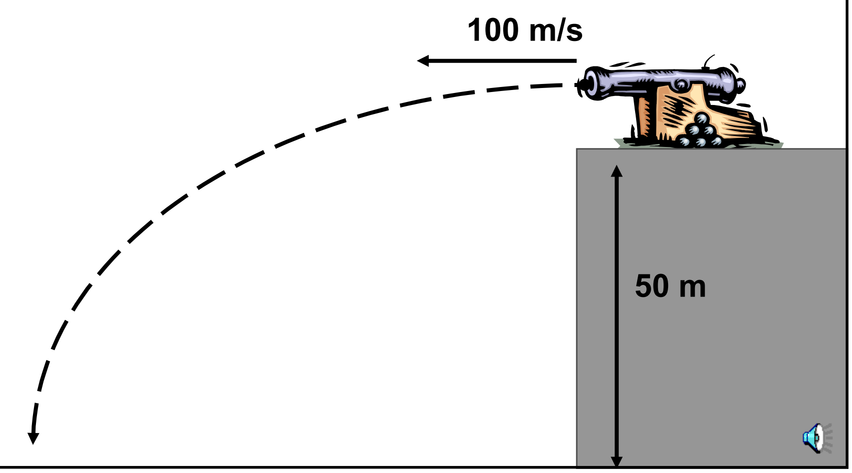
There are no new mathematical formulas for this tutorial topic.

However, all the old formulas from motion in 1 dimension still apply.

32/48

Projectile Example

A cannon fires a ball from the top of a 50 m high cliff. The projectile is fired at 100 m/s. How far away from the base of the cliff will it land?



33/48

Projectile Solution

When only the vertical direction is considered:

- **Its initial vertical velocity is 0 m/s**
- **The distance is -50 m (negative refers to down)**
- **Its acceleration is -9.8 m/s^2**
- **The time in the air, t, is unknown.**
- **We can use one of the previous kinematic formulas.**

“v” refers to the vertical direction

$$d_v = v_{iv}t + \frac{at^2}{2}$$

34/48

Mathematics of Solution

$$d_v = v_{iv}t + \frac{at^2}{2}$$

$$d_v = \frac{at^2}{2}$$

Since v_i =0 m/s, that term drops out.

$$\sqrt{\frac{2d_v}{a}} = t$$

Time isn't the final answer, but it will lead us to the distance.

$$\sqrt{\frac{2(-50m)}{-9.8m/s^2}} = t = 3.2\ \text{sec}$$

35/48

Obtaining the Horizontal Distance

When only the horizontal motion of the projectile is considered:

• Since gravity acts only vertically, there is 0 acceleration.

• Horizontally, the projectile moves with a constant speed.

• We can use one of the previous kinematic formulas.

Constant horizontal velocity.

$$v_h = \frac{d_h}{t}$$

36/48

More Mathematics of Solution

$$v_h = \frac{d_h}{t}$$

Don't confuse this distance with the vertical distance previously used.

$$d_h = t(v_h)$$

$$d_h = (3.2\cancel{s})(100m/\cancel{s})$$

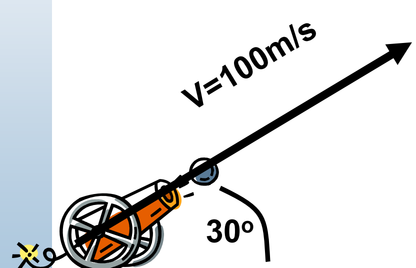
$$d_h = 320m$$

Measured from the base of the cliff.

37/48

Projectiles at an Angle

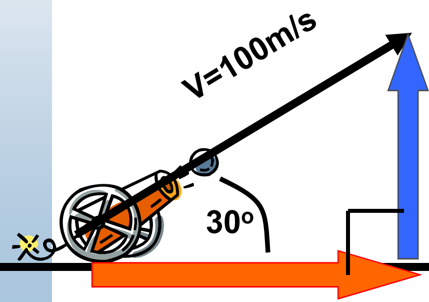
Again a projectile is fired at 100 m/s. This time there is no cliff, but the barrel is elevated at 30° with the ground. How far away will this projectile land?



38/48

Separate into Components

Resolve the diagonal motion of the cannonball into horizontal and vertical components.

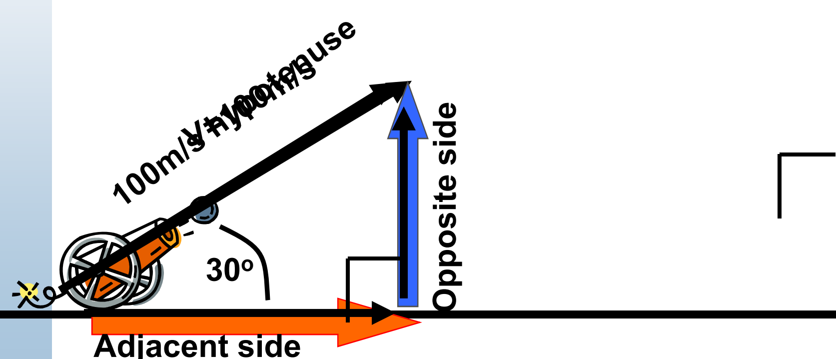


These separate components will be used when motion in only 1 direction is addressed.

39/48

Calculation of Components

Consider the cannonball velocity and its components like the sides of a right triangle. Simple trig functions can be used to calculate all components.



40/48

Component Values

Vertical component:

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin 30^\circ = \frac{opp}{100 m/s}$$

$$opp = .5\,(100 m/s)$$

$$opp = 50 m/s$$

Horizontal component:

$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 30^\circ = \frac{adj}{100 m/s}$$

$$adj = .87\,(100 m/s)$$

$$\mathbf{adj = 87\ m/s}$$

41/48

Calculating Time in Air

We now know that the cannonball initially moves upwards at 50 m/s. (vertical component)

Thus, it will be moving downward at -50 m/s when it returns to the Earth.

Next, use our old acceleration definition.

$$a = \frac{v_f - v_i}{t} \quad \Rightarrow \quad t = \frac{v_f - v_i}{a}$$

$$t = \frac{-50\ m/s - 50\ m/s}{-9.8\ m/s^2} \quad \Rightarrow \quad \mathbf{t = 10\ s}$$

42/48

Calculating Distance Traveled

Finally, since we still have a constant horizontal velocity, we can use our old constant velocity formula again.

$$v_h = \frac{d_h}{t}$$

$$d_h = t(v_h)$$

Notice the horizontal component is used here.

$$d_h = 10.2\cancel{s}(87\ \text{m}/\cancel{s})$$

$$d_h = 887\ \text{m}$$

43/48

Additional Formula

With some algebraic acrobatics, our previous example can be combined into one formula to determine the horizontal range of a projectile:

$$d_h = \frac{v^2 \sin 2\theta}{g}$$

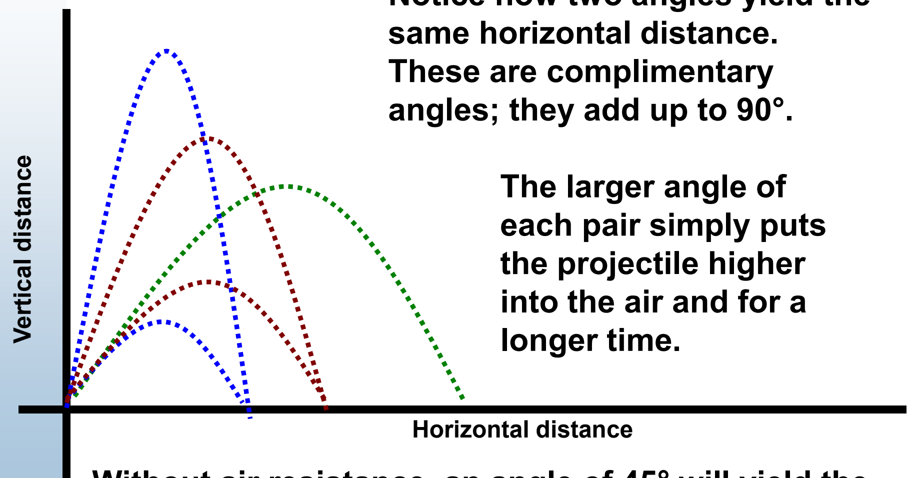
Launch velocity squared

Double the angle of launch measured from the horizon.

Acceleration from gravity, assign as a + here

44/48

Launches at Various Angles



Notice how two angles yield the same horizontal distance. These are complimentary angles; they add up to 90°.

The larger angle of each pair simply puts the projectile higher into the air and for a longer time.

Without air resistance, an angle of 45° will yield the maximum range.

45/48

Learning Summary

- Use the appropriate component when describing motion in a certain direction.
- Vectors have magnitude and direction. Scalars have magnitude only.
- Use old kinematics formulas.
- Add vectors tip to tail. The resultant is the sum of two vectors.
- Split vectors into components using mathematics. (trigonometry)

46/48

Congratulations

You have successfully completed the tutorial

Vectors and Kinematics in 2 Dimensions

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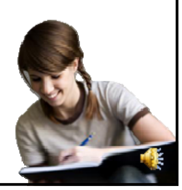
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!



48/48

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