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Teach Yourself Calculus-Based Physics in 24 Hours

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Kinematics in One Dimension

Physics Rapid Learning Series

Wayne Huang, Ph.D.
Keith Duda, M.Ed.
Peddi Prasad, Ph.D.
Gary Zhou, Ph.D.
Michelle Wedemeyer, Ph.D.
Sarah Hedges, Ph.D.

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Learning Objectives

By completing this tutorial, you will:

- Understand and describe constant velocity situations.
- Understand and describe accelerated motion situations.
- Use kinematics equations to solve motion problems.

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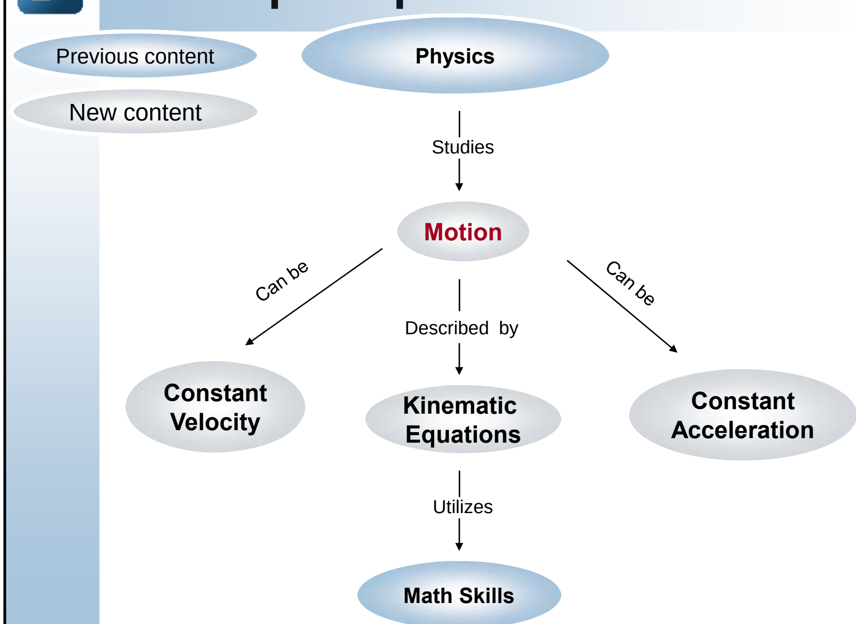
Concept Map



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Basic Concepts

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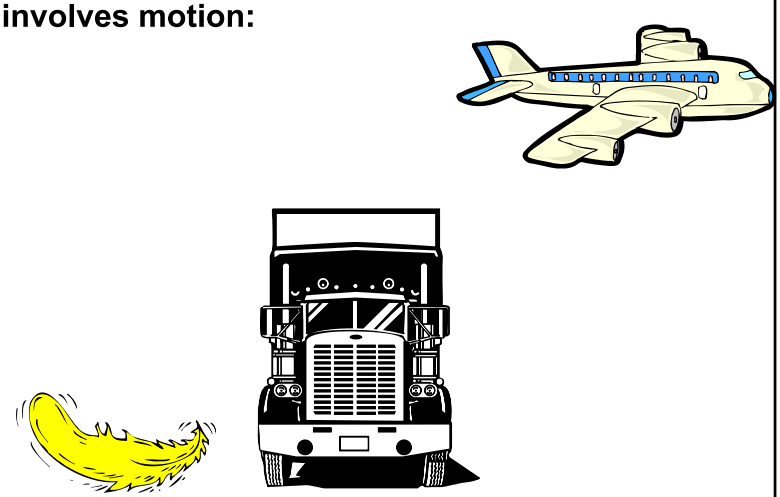
Motion

Almost everything we witness in our everyday lives involves motion:



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Definition - Kinematics

Kinematics – The mathematical description of motion without any reference to the cause.

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Vector Preview

Although vectors will be fully explained in a later tutorial, a brief introduction is needed here:

A vector is a quantity that has magnitude, or size, and direction.

Examples:
Velocity, Displacement, Acceleration, Weight

A scalar is a quantity that has only magnitude, or size.

Examples:
Speed, Distance, Temperature, Mass

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Similarities

Velocity and **speed** are similar terms. They are often used interchangeably. However, there is a difference:

Speed→ Scalar

Velocity→ Vector

Velocity includes a direction with magnitude, and **speed** is only a magnitude.

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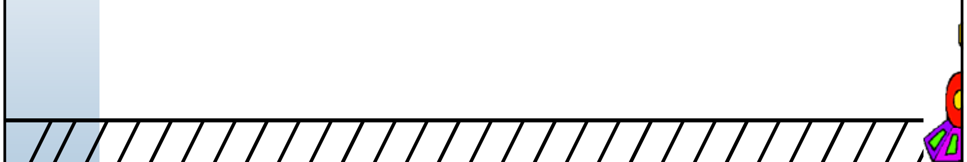
Constant Velocity

Imagine that a toy train is moving forward along the tracks at a constant and steady rate. There is no slowing or speeding in its motion.



This motion represents a **constant velocity.**

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Velocity Formula

The velocity of this train can be found with:

$$v = \frac{d}{t}$$

Velocity, m/s

Distance covered, m

Time elapsed, s

Typical metric units are listed, but others can be used.

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Average Velocity

Sometimes the previous formula is written as:

$$\bar{v} = \frac{d_{total}}{t_{total}}$$

The bar above the v means average. If an object moves with a constant velocity, the average velocity is the same value.

If an object changes its velocity, this formula may be more useful.

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Constant Velocity Example - Problem

If sun light takes about 8 minutes to go from the sun to the earth, how far away from the sun is the earth?
Hint: **light travels at 186,000 miles per second!**

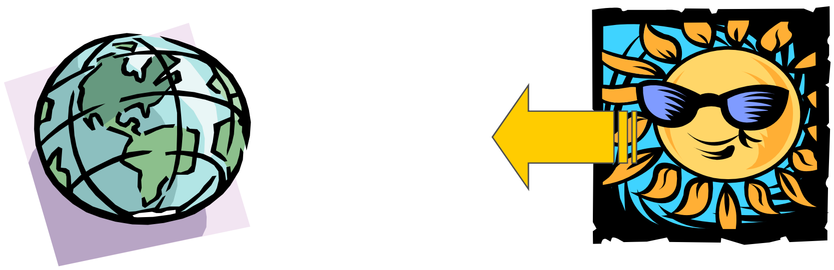

Illustration <u>not</u> to scale

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Constant Velocity Example - Solution

If sun light takes about 8 minutes to go from the sun to the earth, how far away from the sun is the earth?
Hint: **light travels at a constant 186,000 miles per second!**

$$v = \frac{d}{t}$$

Rearrange formula for desired variable

$$d = t\,v$$

$$d = \left(186{,}000\,\frac{mi}{\cancel{sec}}\right) x\ \ 480\cancel{sec}$$

$$d = 8.9\,x10^{7}\ \text{miles}$$

$$8.0\,\text{min}\,x\,\frac{60\text{ sec}}{1\text{min}} = 480\text{sec}$$

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Instantaneous Velocity

Instantaneous velocity: The speed of an object at any particular instant.

The average speed and instantaneous speed may not be the same!

A car that gradually speeds up more and more has an instantaneous speed that always changes.

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Calculus Connection for Velocity

Since velocity can be defined as the change in position per unit of time, it can also be described as:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

This is not "d" for distance. This is the derivative of x with respect to t.

This is not the average velocity. Instead, this is a way of describing the instantaneous velocity as a derivative of position with respect to time.

Note: Here x is used to represent distance or position. This is often done in mathematical discussion.

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Integration of Velocity

If the derivative of position with respect to time is velocity, then the integral of velocity with respect to time must be position.

$$v = \frac{dx}{dt} \qquad x = \int v\,dt$$

Consider the motion of an object moving at a constant velocity. This velocity could be graphed as a function to time.

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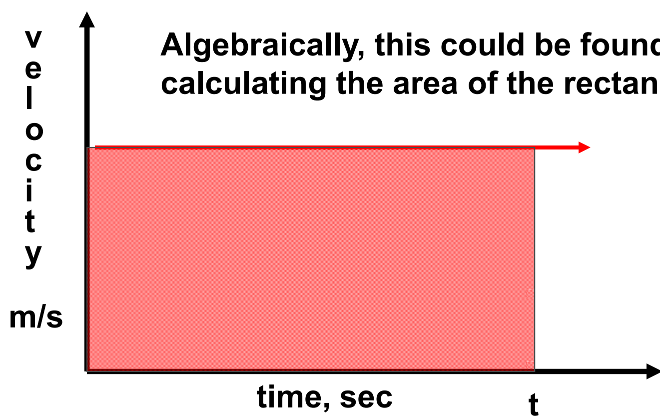
Graphing and Integrating

While moving at a constant velocity, the object will cover a certain distance at time t.

Algebraically, this could be found by calculating the area of the rectangle.



Using calculus: $\text{position} = \int_0^t v\,dt$

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Instantaneous or Average Speed?

If the police monitor your speed with a radar gun, are they checking your instantaneous speed or your average speed?

Instantaneous speed. They would be interested in noticing if you exceed the speed limit at any point in time.

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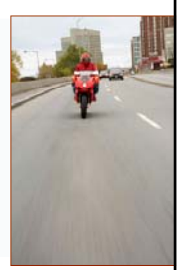
Acceleration



Not all things move at a constant rate. Many will accelerate.

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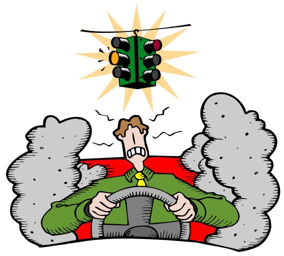
Acceleration – Explained

If you have a difficult car commute to work, you are definitely not moving at a constant rate.



At any one time you may have:

- **a high velocity**
- **an increasing or decreasing velocity**
- **a velocity of 0 m/s**

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Definition - Acceleration

Acceleration: Rate of change of velocity

Acceleration describes how fast an objects speed is changing per amount of time.

If an object has a constant velocity, then its acceleration would be zero.

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Acceleration Formula

$$a = \frac{\Delta v}{\Delta t}$$

Acceleration

Change in velocity, m/s

Sometimes shown as:

$V_{final} - V_{initial}$

or

$V_f - v_i$

Time interval, sec

Δ = Greek letter Delta, change in.

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Calculus Connection for Acceleration

Since acceleration is defined as the change in velocity per change in time, it can also be described as:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

This is not "d" for distance. This is the derivative of v with respect to t.

This means that acceleration is the derivative of velocity.

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Acceleration: Units

Typical metric unit for acceleration:
m/s^2

- Describes the change in velocity each second.
- Read as "meters per second squared"
- Sometimes written as m/s/s

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Real Acceleration Example

During tests to see how much acceleration a human could handle, a person on a rocket powered sled moves at +1020 km/hr. They are quickly brought to a halt in 1.4 seconds. How much acceleration did they experience?

Hint: Find and identify key information

- Final velocity = 0 m/s
- Time interval
- Initial velocity

Acceleration = unknown

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Acceleration Example - Solution

Convert km/hr into m/s

$$\frac{1020\text{km}}{\text{hr}} \times \frac{1\text{hr}}{60\text{min}} \times \frac{1\text{min}}{60\text{ sec}} \times \frac{1000\text{m}}{1\text{km}} = 283.3\text{ m/s}$$

$$a = \frac{\Delta v}{\Delta t} \quad \text{equals} \quad a = \frac{v_f - v_i}{\Delta t}$$

$$a = \frac{0 - 283.3\text{ m/s}}{1.4\text{s}} = -202.4\text{ m/s}^2$$

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Free Fall

On Earth, when an object falls under only the influence of gravity, it speeds up, or accelerates.

Thus, an object doesn't fall at a constant speed. Its speed increases at a constant rate.

It accelerates at -9.8 m/s/s.

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Diving

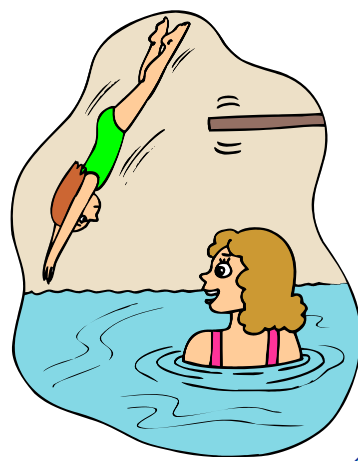
You jump off a very high diving board into a pool. How do you speed up as you fall?

t = 0 sec v = 0 m/s

t = 1 sec v = -9.8 m/s

t = 2 sec v = -19.6 m/s

t = 3 sec v = -29.4 m/s



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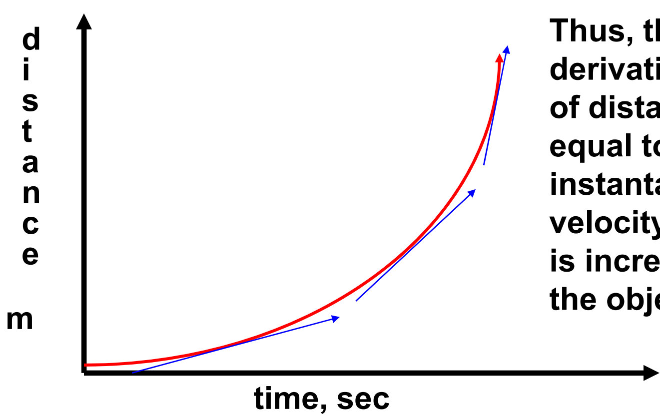
Graphical Description for Velocity

Consider the motion of an accelerating, freely falling object as in the graph below.

At any particular point, the slope of a tangent to the curve represents the instantaneous velocity.



Thus, the derivative/slope of distance is equal to the instantaneous velocity. Also, it is increasing as the object falls.

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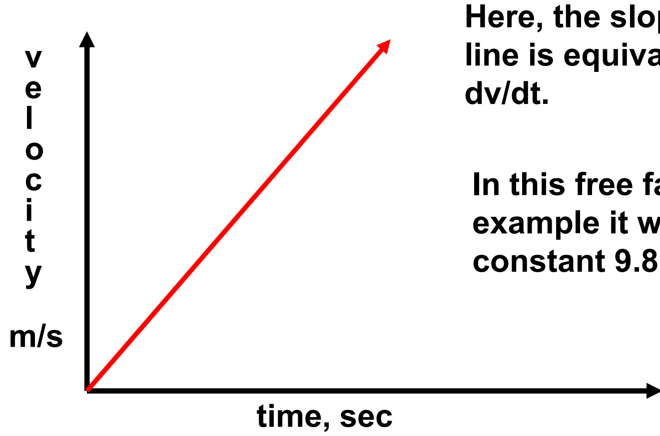
Graphical Description for Acceleration

Again consider a freely falling accelerating body. This time look at a graph of velocity vs. time.

Notice how the velocity increases at a constant rate.



Here, the slope of the line is equivalent to dv/dt.

In this free fall example it would be a constant 9.8m/s/s.

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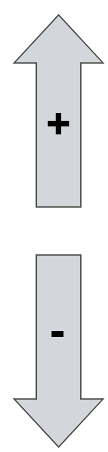
Direction/Sign Conventions

Upward is often considered the positive direction.

Downward is usually considered the negative direction.

Thus, the acceleration from gravity is usually given a negative sign.

$g = -9.8 \text{ m/s}^2$



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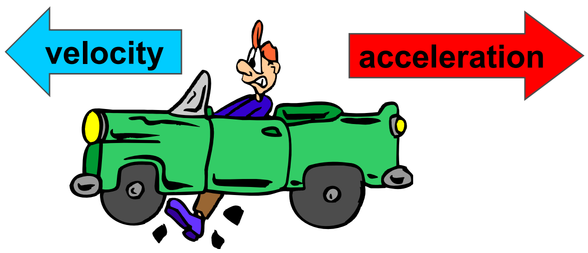
Additional Sign Conventions

When motion occurs horizontally, either direction could be arbitrarily considered +.

However, throughout the problem, that direction must always be kept +, and the opposite direction must be -.



We could say that the car braking to a stop has a + velocity to the left, but a – acceleration to the right.

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How Far?

How fast you speed up doesn't necessarily indicate how much farther you will go.

The distance you cover is proportional to the *square* of the time:

$$d = v_i t + \frac{at^2}{2}$$

Time, s

Distance, m

Initial velocity, m/s

Acceleration m/s²

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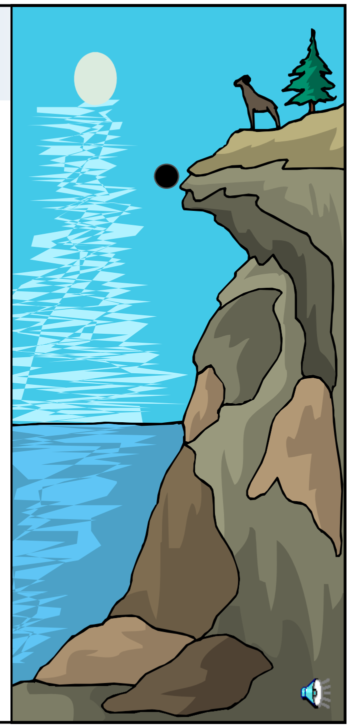
Falling How Far?

If a stone falls from rest off a cliff for 1 sec, how far does it fall? For 2 sec?

Hint: Find and identify key information

- Initial velocity = 0 m/s (rest)
- Acceleration from gravity = $-9.8\ m/s^2$
- Time = 1sec, then 2 sec
- Distance is unknown



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Falling How Far?

$$d = v_i t + \frac{at^2}{2}$$

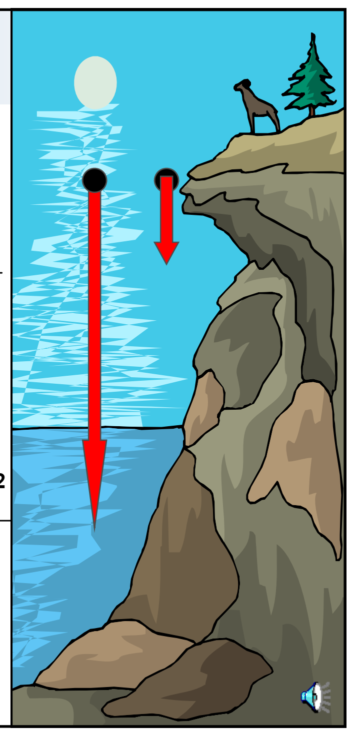
$$d = (0m/s)1sec + \frac{-9.8m/s^2(1s)^2}{2}$$

$$d = -4.9m$$

$$d = (0m/s)2sec + \frac{-9.8m/s^2(2s)^2}{2}$$

$$d = -19.6m$$

Notice the difference!



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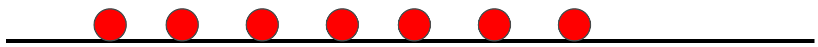
Free Fall - Explained

If you could take a snapshot each second of a ball rolling along a table at a constant velocity, it might look like this:



If you could take a snapshot each second of a ball rolling along a table with a constant acceleration, it might look like this:

Free Fall - Mass

You might notice that some objects don't seem to fall as fast as others. Ex: rock, feather

None of the previous equations incorporate mass as a variable. Thus, the mass of an object shouldn't effect how it falls.

In the absence of air resistance, a rock and a feather would fall at exactly the same rate. Where could you accomplish this?

The moon!

Additional Formula

With some algebra, the previous formulas can be combined into one additional one that can sometimes be useful when time isn't known:

$$v_f^2 = v_i^2 + 2ad$$

- v_f: Final velocity, m/s
- v_i: Initial velocity, m/s
- a: Acceleration m/s^2
- d: Distance, m

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Problem-Solving Example

If a model rocket was fired straight up with a velocity of 200m/s, how high should it rise before beginning to fall back to Earth?

Hint: Find and identify key information

- Initial velocity = +200 m/s
- Final velocity = 0 m/s (stops before it falls to Earth)
- Acceleration from gravity = -9.8 m/s^2
- Time =?
- Distance=?

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Problem-Solving Example (2)

$$v_f^2 = v_i^2 + 2ad$$
$$-v_i^2 \quad -v_i^2$$

$$\frac{v_f^2 - v_i^2}{2a} = \frac{2ad}{2a}$$

$$\frac{v_f^2 - v_i^2}{2a} = d$$

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Problem-Solving Example (3)

$$d = \frac{(0m/s)^2 - (200m/s)^2}{2(-9.8m/s^2)}$$

V_f of zero makes math easier

$$d = \frac{-(40000m^2/s^2)}{(-19.6m/s^2)}$$

Notice how units cancel leaving just meters.

$$d = 2040m$$

The + sign for our answer makes sense since we assigned the original direction, up, as + too.

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KUDOS – The Problem Solving Method

1. Step K: Identify the Known information in the problem, along with the unknown quantity.

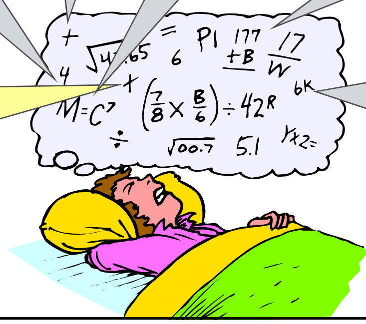
2. Step U: Identify the Unknown to resolve (x).

3. Step D: Define a formula based on what you know, and what you're looking to find.

4. Step O: Output the result by doing algebra and computing carefully.

5. Step S: Substantiate the answer by checking the units, significant figures etc.

In some texts, "x" may be used instead of the variable "d". They are equivalent.



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Calculus Example

Sometimes accelerations are not constant as we have previously assumed. If the position/distance of a particle along the x axis is given by the function: $x(t)=8+9t-2t^3$, what will the acceleration be at t=3 seconds?

- Recall that velocity is defined as dx/dt.
- Then remember that acceleration is defined as dv/dt.
- To get our mathematical relationship for acceleration, we must take the second derivative of the given position function.

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Calculus Example Solution

$$x(t) = 8 + 9t - 2t^3$$

The units have been omitted for clarity, but they would be typical SI units.

$$\frac{dx}{dt} = 9 - 6t^2 = \text{velocity}$$

$$\frac{dv}{dt} = -12t = \text{acceleration}$$

Substituting t = 3 gives $-36 m/s^2$

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Learning Summary

$v_f^2 = v_i^2 + 2ad$
Time doesn't matter

Constant velocity:

V = d/t

Units:

t → sec
d → m
v → m/s
a → m/s^2

Acceleration = changing velocity

A=Δv/Δt

Distance travelled depends on both velocity, acceleration and time:

$$d = v_i t + \frac{at^2}{2}$$

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Congratulations

You have successfully completed the tutorial

Describing Motion: Kinematics in One Dimension

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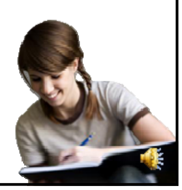
What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!



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