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1/77

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Plane Geometry and Spatial Thinking

SAT Rapid Learning Series

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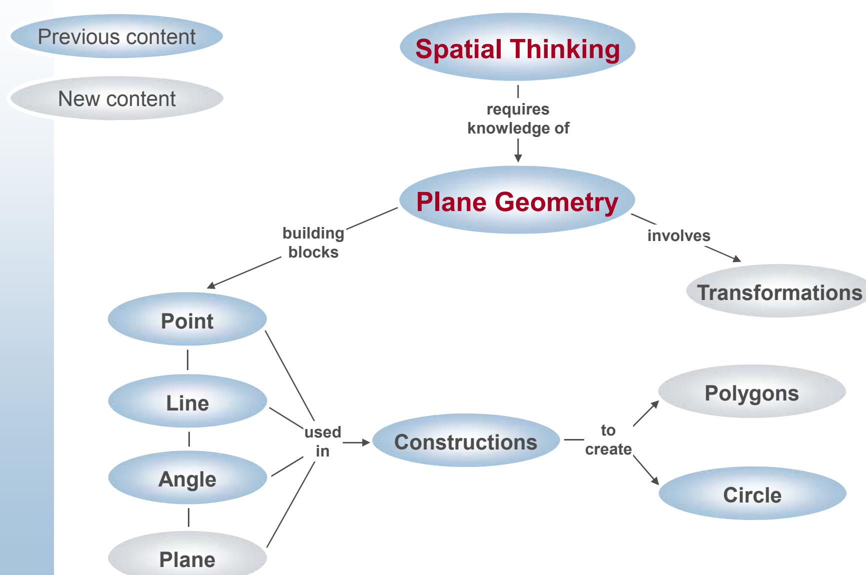
Learning Objectives

After completing this tutorial, you will be able to:

- Define point, line, plane and angle
- Define polygon and circle
- Construct geometric figures
- Perform transformations on polygons
- Use diagrams to solve problems

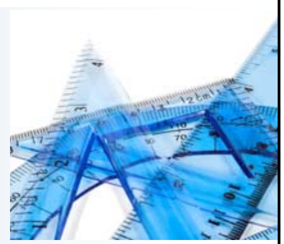
3/77

Concept Map



4/77

Plane Geometry Basics



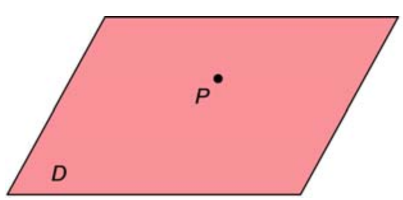
5/77

Definition: Point

Point – The result of the intersection of two lines; represented by a dot and labeled with a capital letter.

Point is the main building element of any figure in geometry. Any figure is the set of an infinite number of points.

A point has no dimensions; it has no length, no width, and no thickness.



6/77

Definition: Plane Geometry

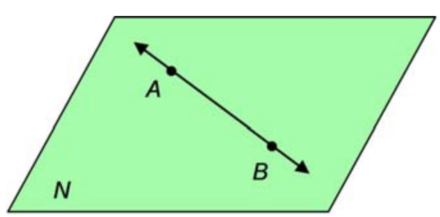
Plane geometry – The science of measurement; the geometry dealing with figures in a plane.

7/77

Definition: Line

Line – A figure formed by connecting two points and extending beyond each point in both directions; represented with arrows at each end.

A line is formed when an infinite number of points lie next to each other in a straight path.

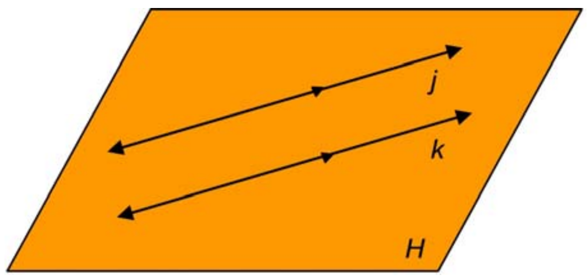


8/77

Definition: Parallel Lines

Parallel lines – Two or more lines in the same plane that do not intersect.

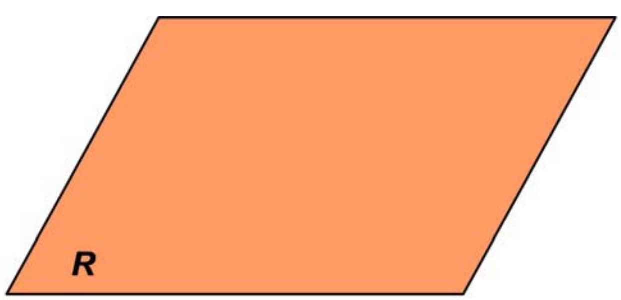
Parallel lines share no common points.



9/77

Definition: Plane

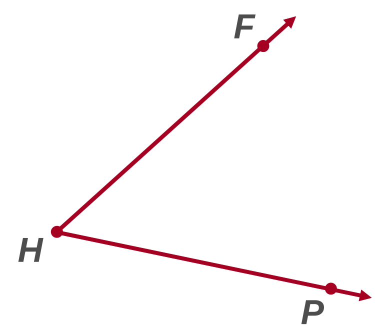
Plane – A flat surface that extends indefinitely in all directions; represented by a parallelogram.



10/77

Definition: Angle

Angle – A figure formed by two rays with a common initial point.



11/77

Classification of Angles

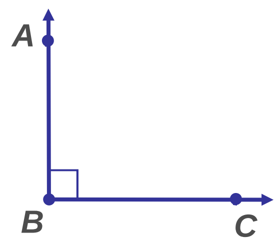
Angles can be classified by their measures as one of the following:

- **Acute**
- **Right**
- **Obtuse**
- **Straight**

12/77

Definition: Right Angle

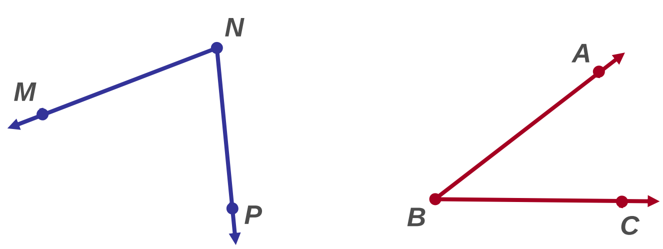
Right angle – An angle with a measure of exactly 90°.



13/77

Definition: Acute Angle

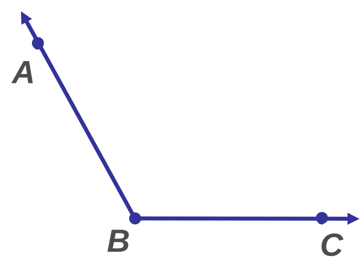
Acute angle – A positive angle with a measure less than 90°.



14/77

Definition: Obtuse Angle

Obtuse angle – An angle with a measure between 90° and 180°.



15/77

Definition: Straight Angle

Straight angle – An angle with a measure of exactly 180°.

16/77

Example: Classifying Angles

What type of angle is represented by the corners of a regular sheet of paper?

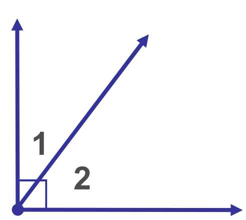
- [ ] acute
- [ ] obtuse
- [x] right
- [ ] straight

17/77

Angle Pair Relationships

Two angles can have the following relationships:

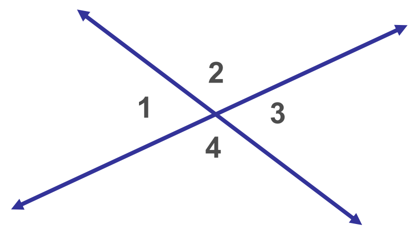
- Vertical
- Complementary
- Supplementary
- Adjacent



18/77

Definition: Vertical Angles

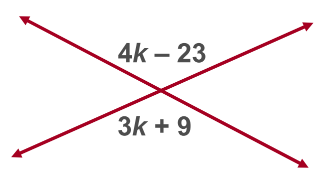
Vertical angles – Two angles that are across from each other at the intersection of two lines; they are always congruent.



19/77

Example: Vertical Angles

Using the vertical angles below, find the value of *k*.



Solution:

$$4k - 23 = 3k + 9$$

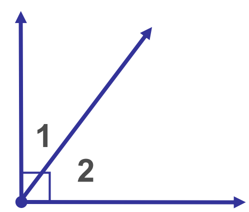
$$k - 23 = 9$$

$$k = 32$$

20/77

Definition: Complementary Angles

Complementary angles – Two angles whose sum is 90°.



21/77

Example: Complementary Angles

∠1 and ∠3 are complementary angles.
$m\angle 3 = 62°$. Find $m\angle 1$.

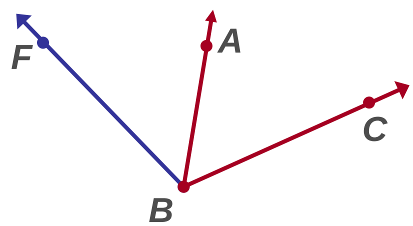
Solution:

$$m\angle 1 + m\angle 3 = 90°$$
$$m\angle 1 + 62° = 90°$$
$$m\angle 1 = 28°$$

22/77

Definition: Adjacent Angles

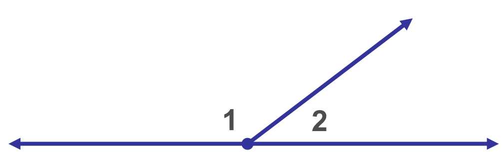
Adjacent angles – Two angles that share a common side and a common vertex, but do not overlap.



23/77

Definition: Supplementary Angles

Supplementary angles – Two angles whose sum is 180°.

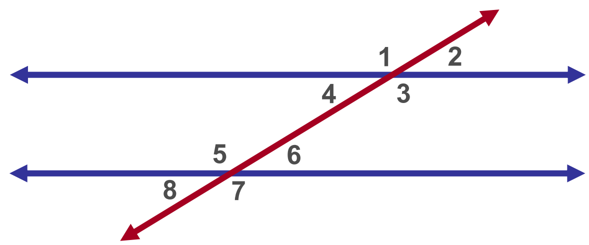


24/77

Parallel Lines and Transversals

Two coplanar lines are either intersecting or parallel.

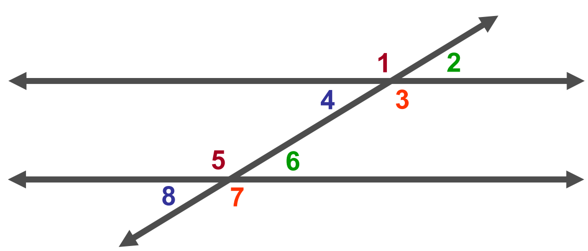
When two parallel lines are intersected by a transversal, eight angles are formed around the points of intersection.



25/77

Definition: Corresponding Angles

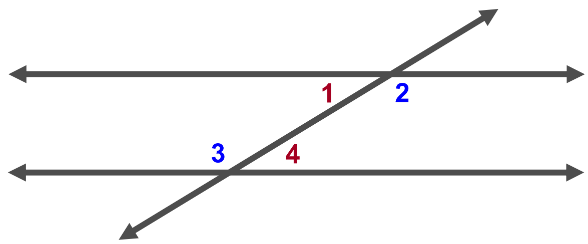
Corresponding angles – Pairs of angles that are positioned the same at the intersection of two parallel lines and a transversal; they are congruent.



26/77

Definition: Alternate Interior Angles

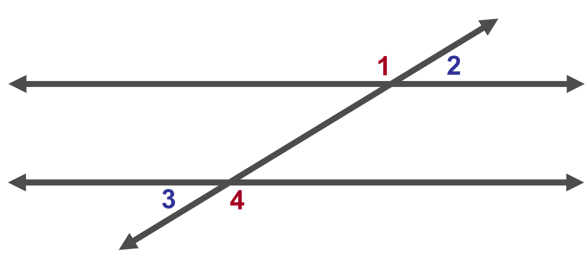
Alternate interior angles – Pairs of angles located between the parallel lines on opposite sides of the transversal; they are congruent.



27/77

Definition: Alternate Exterior Angles

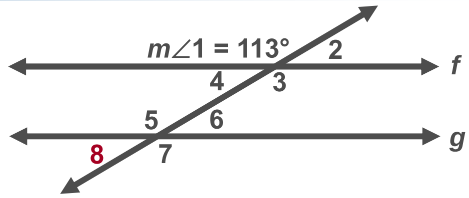
Alternate exterior angles – Pairs of angles located outside the parallel lines on opposite sides of the transversal; they are congruent.



28/77

Example: Parallel Lines

Lines *f* and *g* are parallel. Find $m\angle 8$.



Solution:

$$\angle 1 \cong \angle 5$$

$$m\angle 5 = 113°$$

$$m\angle 5 + m\angle 8 = 180°$$

$$113° + m\angle 8 = 180°$$

$$m\angle 8 = 67°$$

29/77

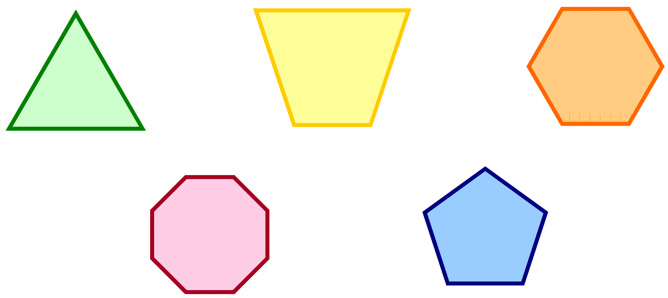
Polygons & Circles

30/77

Definition: Polygon

Polygon – A closed plane figure with three or more sides; each side is a line segment.

Examples:

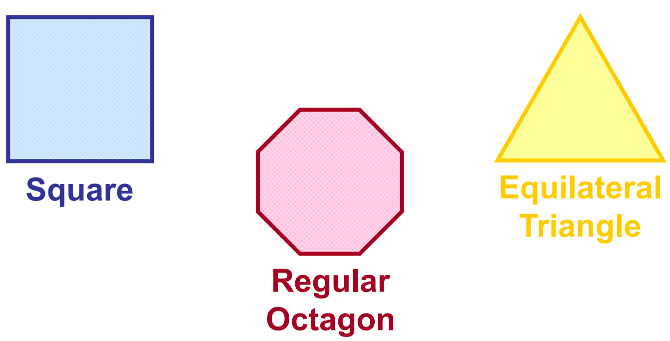


31/77

Definition: Regular Polygon

Regular polygon – A polygon where all sides are congruent and all angles are congruent.

Examples:

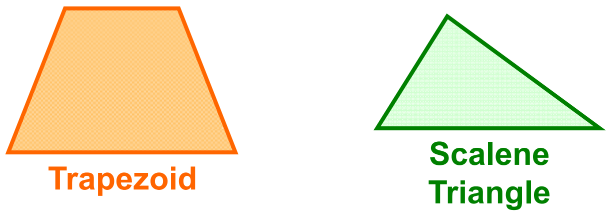


32/77

Definition: Irregular Polygon

Irregular polygon – A polygon where all the sides and angles are not congruent.

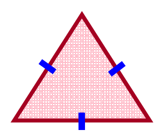
Examples:



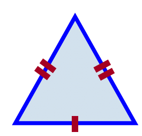
33/77

Classifying Triangles

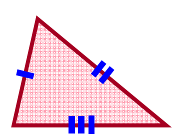
Equilateral triangle – A triangle with all sides congruent; also called equiangular.



Isosceles triangle – A triangle with at least two sides congruent.



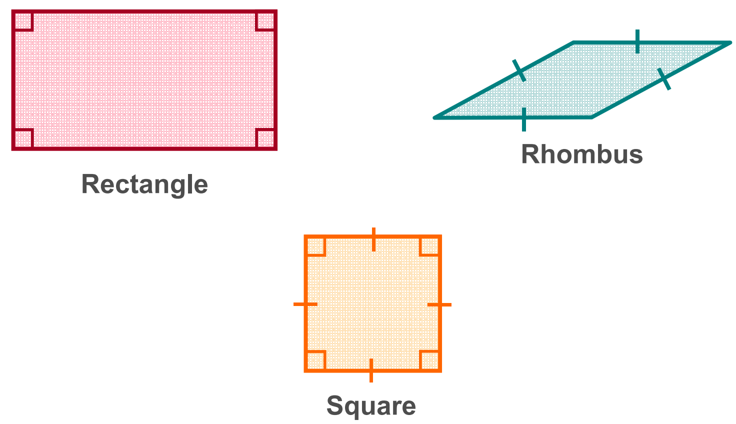
Scalene triangle – A triangle with no two congruent sides; all interior angles have different measures.



34/77

Definition: Parallelogram

Parallelogram – A quadrilateral with two pairs of parallel sides.

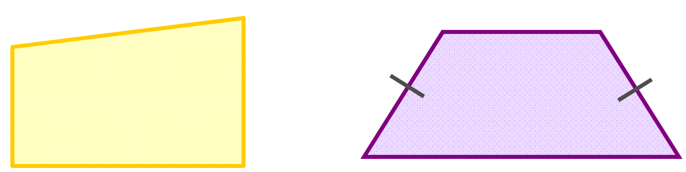


35/77

Definition: Trapezoid

Trapezoid – A quadrilateral with one pair of parallel sides.

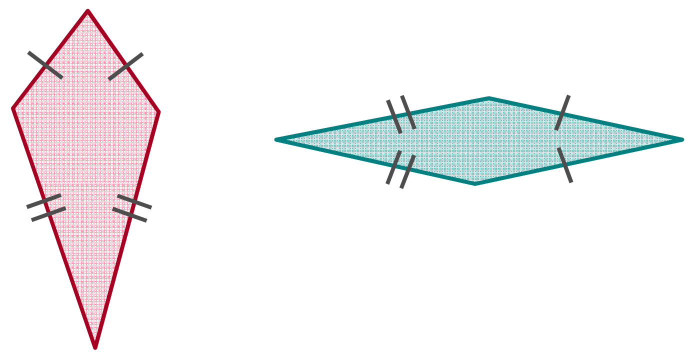
Isosceles trapezoid – A trapezoid with congruent legs.



36/77

Definition: Kite

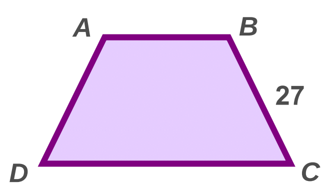
Kite – A quadrilateral with no parallel sides and two pairs of adjacent sides that are congruent.



37/77

Example: Isosceles Trapezoid Legs

Given *ABCD* is an isosceles trapezoid and *BC* = 27, find the length of $\overline{AD}$.



Solution:

BC = 27

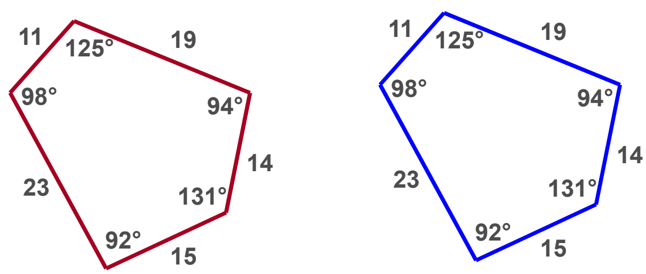
BC = *AD*

AD = 27

38/77

Congruent Polygons

Two figures are congruent if their corresponding sides and angles are congruent.

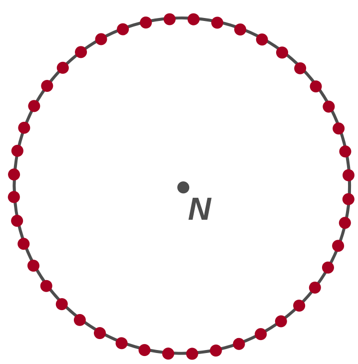


39/77

Definition: Circle

Circle – A set of points that are a fixed distance from a given point (center).

Congruent circles have congruent radii.



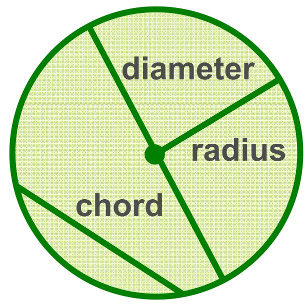
40/77

Circle Components

Chord – A line segment with both endpoints on the circle.

Radius – Any segment that connects a point on a circle to the center of the circle; all radii of a circle have the same length.

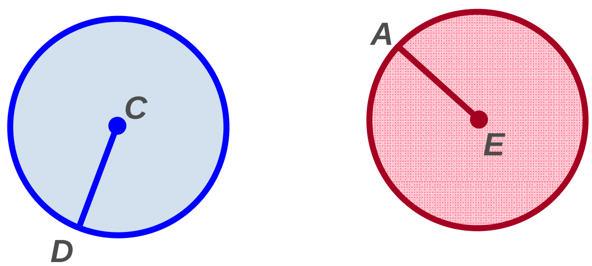
Diameter – Any segment that connects two points on a circle and passes through the center.



41/77

Definition: Congruent Circles

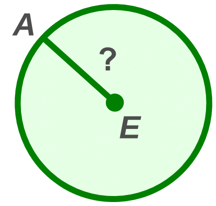
Congruent circles – Two circles that have radii or diameters of the same length.



42/77

Example: Congruent Circles

⊙*C* is congruent to ⊙*E*. The diameter *d* of ⊙*C* is 8 inches. What is the radius *r* of ⊙*E*?



Solution:

⊙*C* ≅ ⊙*E*

diameter of ⊙*E* = 8 in.

radius of ⊙*E* = ½ · diameter

= ½ · 8

= **4 inches**

43/77

Constructions

44/77

Tools of Construction

Rulers and protractors are not used in geometric constructions.

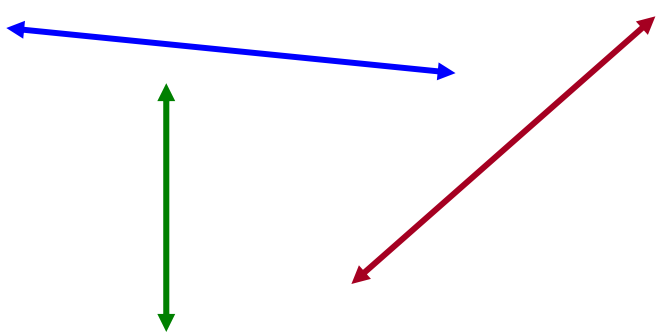
The only tools used when constructing geometric figures are:

- **Straightedge**
- **Compass**

45/77

Definition: Straightedge

Straightedge – Any object that can be used to draw a straight line; not used to measure length.



46/77

Definition: Compass

Compass – A tool used to draw circles and arcs of circles.

A compass has two arms joined at one end and free at the other. One arm is a pointer and the other arm has a pencil attached to it.

47/77

Duplicating Segments – 1

Duplicate $\overline{AB}$.

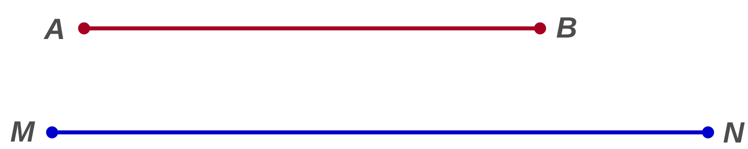
48/77

Duplicating Segments – 2

To construct a segment equal to $\overline{AB}$, draw an arbitrary segment, $\overline{MN}$, using a straightedge.

Make the length of $\overline{MN}$ longer than the length of $\overline{AB}$.

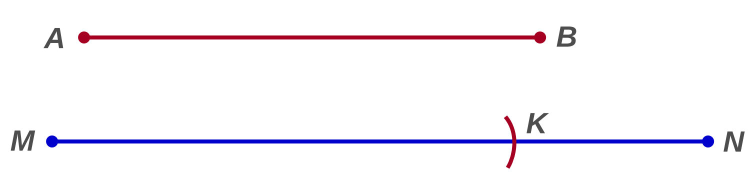
Open a compass and place its pointer at *A* and its pencil at *B*.



Duplicating Segments – 3

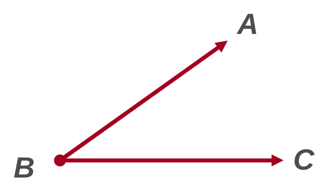
Keeping the compass with the same opening, place its pointer at *M* and draw an arc on $\overline{MN}$.

This arc intersects $\overline{MN}$ at *K*. $\overline{MK}$ is congruent to $\overline{AB}$.



Duplicating Angles – 1

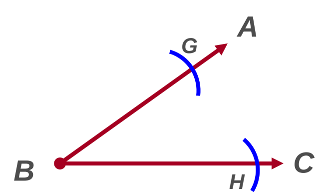
Draw an angle congruent to $\angle ABC$.



51/77

Duplicating Angles – 2

Using a compass, draw two different arcs on $\overrightarrow{BC}$ and $\overrightarrow{BA}$ both centered at *B*. Denote the points of intersection by *G* and *H*, respectively.



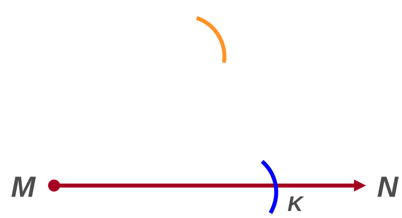
52/77

Duplicating Angles – 3

Draw arbitrary $\overrightarrow{MN}$. Using the compass, draw an arc centered at M whose radius is equal to $\overline{BH}$.

53/77

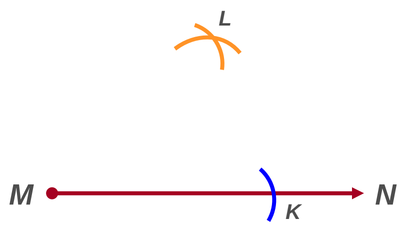
Duplicating Angles – 4



Set the compass radius equal to $\overline{BG}$. Place the pointer at M and draw an arc above $\overrightarrow{MN}$.

54/77

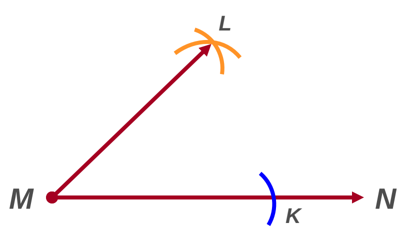
Duplicating Angles – 5



Open the compass as wide as the length of $\overline{GH}$. Place the pointer at *K* and draw an arc. This arc intersects the top arc at *L*.

55/77

Duplicating Angles – 6

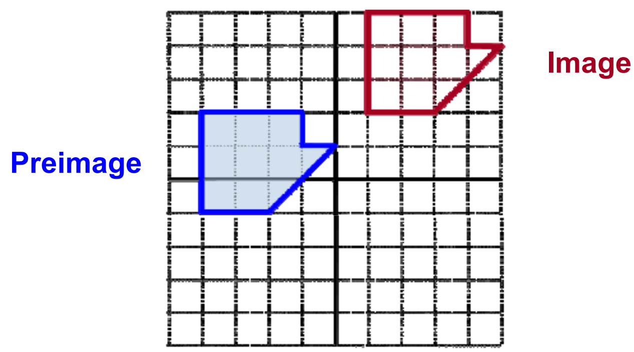


Draw a line through the points *M* and *L*. The result is ∠*LMN*, which is congruent to ∠*ABC*.

56/77

Definition: Transformation

Transformation – A change in position, shape, or size of a figure.



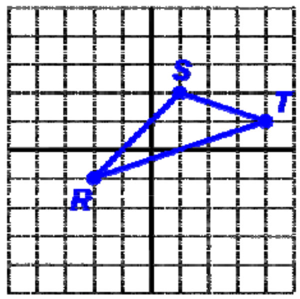
57/77

Transformations

58/77

Preimage Matrix

A matrix can be used to represent the preimage of a translation.

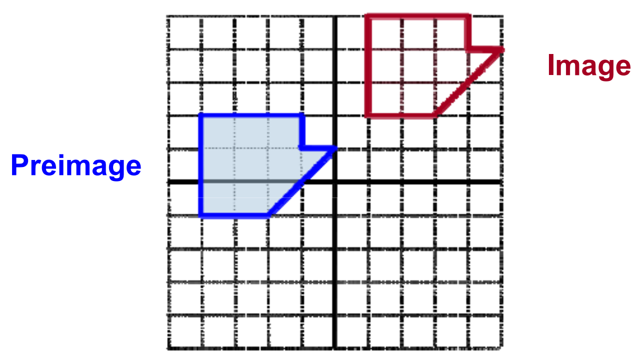


$$\begin{array}{l} \\ x\text{-coordinate} \\ y\text{-coordinate} \end{array} \begin{array}{c} \begin{array}{ccc} R & S & T \end{array} \\ \begin{bmatrix} -2 & 1 & 4 \\ -1 & 2 & 1 \end{bmatrix} \end{array}$$

59/77

Definition: Translation

Translation – a transformation that slides a figure to another location without any change in size or orientation.

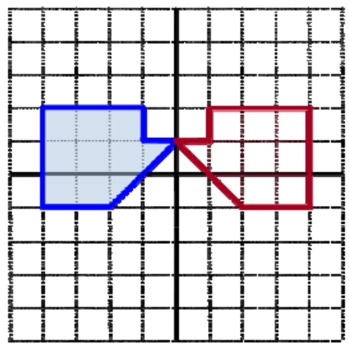


60/77

Definition: Reflection

Reflection – A transformation that flips a figure over a line.

Preimage



Image

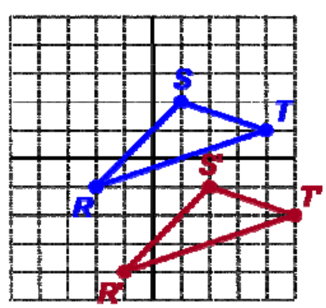
61/77

Translation Matrix

Matrix addition can be used to find the image of a translated figure.

Example:

- Add 1 to each *x*-coordinate.
- Add -3 to each *y*-coordinate.



Preimage — Translation — Image

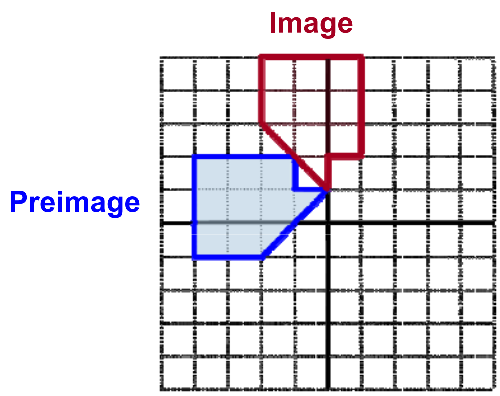
$$\begin{bmatrix} -2 & 1 & 4 \\ -1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ -3 & -3 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 5 \\ -4 & -1 & -2 \end{bmatrix}$$

R S T — R′ S′ T′

62/77

Definition: Rotation

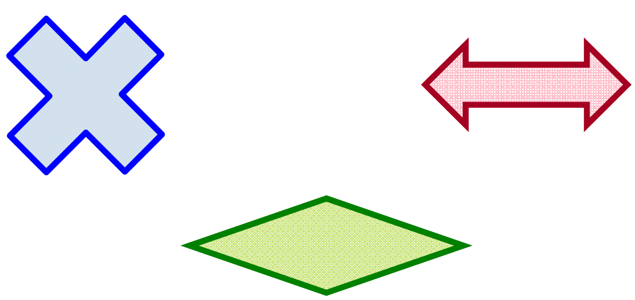
Rotation – A transformation that turns a figure around a fixed point.



63/77

Point Symmetry

If a figure can be mapped onto itself with a 180° rotation, then it has **point symmetry**.



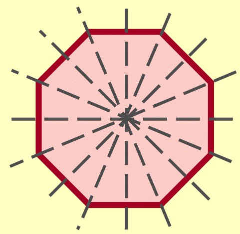
64/77

Example: Reflection Symmetry

How many lines of symmetry does a stop sign have?

Solution:

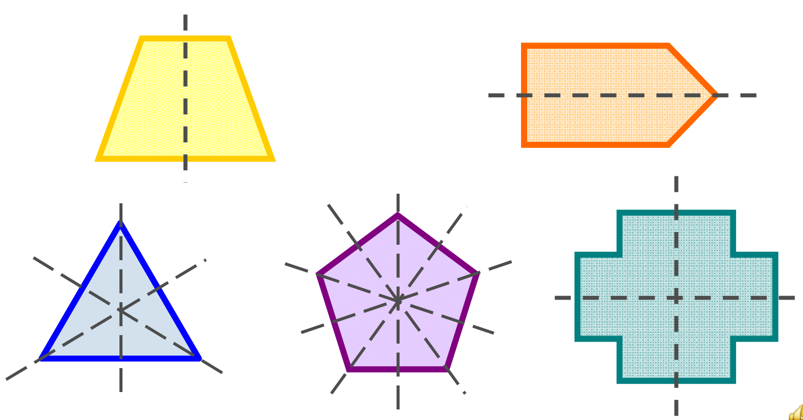
- 4 through the vertices
- 4 through the sides
- **8 lines of symmetry**



65/77

Reflection Symmetry

If a figure can be folded in half so that the halves match exactly, then it has **reflection symmetry**.



66/77

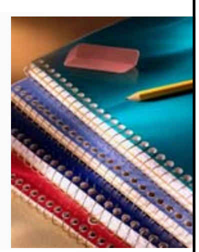
Definition: Tessellate

Tessellate – To cover a plane with identical shapes with no overlapping or gaps.

This tessellation of a trapezoid was created using a combination of transformations.

67/77

Proof in Geometry



68/77

Reasoning with Diagrams

Some problem situations are easier to understand when a related diagram is given.

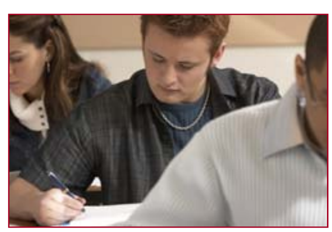
Diagrams can be used to list known information and establish what information is needed to solve the problem.

69/77

Reasoning Strategies

Inductive reasoning – A reasoning strategy that uses a set of examples to find a pattern to support a conclusion.

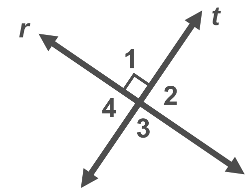
Deductive reasoning – A reasoning strategy that uses facts and theorems to arrive at a conclusion.



70/77

Example 1: Proof Using a Diagram

Prove ∠2, ∠3, and ∠4 are right angles.



Solution:

∠1 ≅ ∠3 → *m*∠3 = 90°

m∠1 + *m*∠2 = 180°

90° + *m*∠2 = 180°

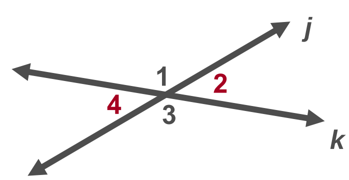
m∠2 = 90°

∠2 ≅ ∠4 → *m*∠4 = 90°

71/77

Example 2: Proof Using a Diagram

Prove ∠2 ≅ ∠4.



Solution:

m∠2 + *m*∠3 = 180°   *m*∠3 + *m*∠4 = 180°

m∠2 + *m*∠3 = *m*∠3 + *m*∠4

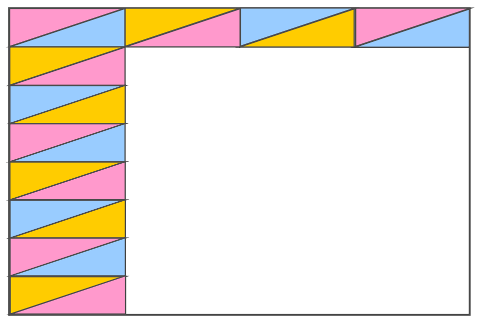
m∠2 = *m*∠4

∠2 ≅ ∠4

72/77

Example 3: Proof Using a Diagram – 1

Justin is tiling an 8 in. × 12 in. surface with colored glass tiles. Each tile is a right triangle with a base of 1 in. and a height of 3 in. How many tiles will Justin need to tessellate (cover) the surface?

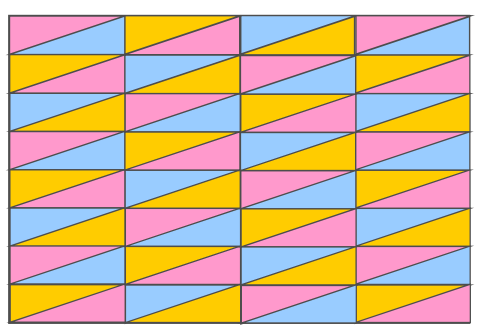


Row = 8 triangles **Column = 8 rows**

73/77

Example 3: Proof Using a Diagram – 2

Justin is tiling an 8 in. × 12 in. surface with colored glass tiles. Each tile is a right triangle with a base of 1 in. and a height of 3 in. How many tiles will Justin need to tessellate (cover) the surface?



8 triangles × 8 rows = 64 triangles

74/77

Learning Summary

Rulers and **protractors** are not used in geometric constructions.

A **point** has no dimensions: no length, width, or thickness.

A **transformation** is a change in the position, shape, or size of a figure.

Parallel lines never intersect and share no common points.

Inductive reasoning uses examples to draw conclusions; **deductive reasoning** uses facts.

75/77

Congratulations

You have successfully completed the core tutorial

Plane Geometry and Spatial Thinking

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What's Next …

Step 1: Concepts – Core Tutorial (Just Completed)

→ **Step 2: Practice – Interactive Problem Drill**

Step 3: Recap – Super Review Cheat Sheet

Go for it!

77/77

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